# Two-dimensional Location Algorithm Based on Time Difference of Arrival 

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#### Abstract

: Service robots, driverless cars and unmanned aerial vehicles gradually appear in people's life with the development of new technologies such as internet of things, big data, cloud computing and artificial intelligence. These applications rely on location technology. In order to meet the location requirements, it is necessary to realize two-dimensional location, to improve the locating accuracy and to reduce the locating error through relevant processing. In this paper, we study two-dimensional location algorithm based on time difference of arrival. At first, the mathematical basis and principle of the location algorithm are analyzed. Then, the location algorithm is simulated based on MATLAB to verify the algorithm. At last, the location algorithm is deeply studied and the optimization scheme is proposed. The research has important reference value for the practical application of location algorithm based on TDOA.


Keywords: Time difference of arrival (TDOA); Ultra wide band (UWB); Location algorithm; Locating accuracy; Locating error.

## I. INTRODUCTION

### 1.1 Background of Location Technology Research

New industries such as smart home, smart medicine, smart security, smart logistics, smart power, intelligent agriculture, intelligent transportation, intelligent city and intelligent campus continue to emerge with the rapid development of new technologies such as Internet of Things, Big Data, Cloud Computing and Artificial Intelligence. Traditional industries and enterprises have been rapidly upgraded or transformed with the strong support and promotion of national policies. The new generation of science and technology is surging forward with a large inflow of talents and capital.

As an indispensable key link in emerging industries, location technology can process the rich data in the virtual information space, obtain the location information of people and things in real world, and make offline people and things be searched, located and connected like online information, so as to break the boundary between real world and virtual world and truly realize the interconnection of all things.

Since the Federal Communications Commission (FCC) issued the E-911 location requirements in 1996, many countries have actively carried out research on mobile station location technology [1]. In recent years, IEEE journals have disclosed many invention patents of location technology, and some companies specializing in location technology research and development have emerged in the world.

### 1.2 Situation of Location Technology Research

According to the way that anchor nodes obtain information, location technology can be divided into two kinds: range based location technology and range free location technology. Range based location technologies include RSSI, TOA, TDOA and AOA etc., and range free location technologies include convex planning, MDS-MAP, DV-Hop etc.

Huangbao Liu proposed a centroid location algorithm based on RSSI. Firstly, the distances from the node to be located to the surrounding anchor nodes are determined by the size of RSSI, and the three anchor nodes closest to the node to be located are found. The radiation range of the node to be located is divided into six areas by using the vertical bisector of the three sides of the triangle determined by the three anchor nodes. Then, the smaller area which contains the node to be located is determined according to RSSI. Finally, the centroid of the area is taken as the location of the node to be located [2].

Doherty proposed a convex programming location algorithm. The communication connection of nodes in wireless sensor networks is regarded as geometric constraint of node location, and the whole network is modeled as a convex set, so the node location problem is transformed into a convex constraint optimization problem, then a globally optimized location scheme is obtained by using semi definite programming and linear programming methods [3].

MDS-MAP is a centralized location algorithm, which can be used for range based locating or range free locating, and can realize relative locating or absolute locating according to the need of practical application. Firstly, generate a global network topology connecting graph, and each edge in the graph is given a distance value. Then, use the shortest path algorithm (such as Dijkstra or Floyd algorithm) to generate the node distance matrix and form the two-dimensional or three-dimensional relative coordinate system of the whole network. When there are many anchor nodes, it is necessary to convert the relative coordinate system into the absolute coordinate system. The experimental results show that in a network with 200 nodes and an average connectivity of 12.1 , the locating error is about $30 \%$ in range based locating and $16 \%$ in range free locating [4].

From the situation of location algorithm research, it can be seen that location technology is not limited to a fixed way, it should comply with the requirements of practical applications and adopt targeted location algorithm according to specific application scenes. Each location algorithm or location system is used to solve specific practical problem and support different application. These algorithms have their own characteristics in many aspects, such as spatial dimension, location area, scene complexity, equipment cost and so on [5-7].

### 1.3 Significance of Location Technology Research

Smart city has become reality from concept. People gradually realize its huge commercial value and begin to enjoy its convenience. For smart city, there are many application scenes that need location. For example, bus location, warehousing and logistics forklift location, UAV location, supermarket personnel location, home elderly location, etc. Different application scene needs special location algorithm. At the same time, we should consider reducing locating error and controlling the cost of location equipment.

This paper introduces the principle of two-dimensional location algorithm, verifies the algorithm by simulation on MATLAB, analyzes the factors that may lead to locating error, and explores the idea of reducing locating error. The work has important reference value for practical application of the algorithm.

### 1.4 Contents of Location Technology Research

Based on TDOA technology and UWB communication equipment, this paper proposes two-dimensional location algorithm under the condition that the coordinates of anchor nodes and the distance from each anchor node to the node to be located (Tag) are known. Simulate on MATLAB to verify the algorithm. On the basis of the realization of location function, put forward the idea to reduce locating error and improve locating accuracy.

## II. RELEVANT KNOWLEDGE OF LOCATION SYSTEM DEVELOPMENT

### 2.1 UWB Pulse Locator

UWB is a carrier free communication technology, which uses nanosecond to microsecond non sine wave narrow pulses to transmit data. By transmitting extremely low-power signals over a wide spectrum, UWB can achieve data transmission rates of hundreds of Mbps to several Gbps in a range of about 10 meters [8]. Compared with the current popular short-range wireless communication technology, UWB has advantages such as high transmission rate, large system capacity, strong anti-interference performance, good confidentiality, very low transmission power, long time of power supply, and little impact of electromagnetic wave radiation on human body, etc. The practical application test results have proved that the UWB system can provide reliable data transmission of more than 480 Mbps indoors under very low power. At the same time, because the narrow pulse has high locating accuracy, it is easy to combine communication and location by using UWB radio communication technology to design and manufacture UWB pulse locator, which is difficult to do by other wireless communication technologies [9].

Compared with GPS, UWB pulse locator has obvious advantages. Firstly, the electromagnetic wave in UWB band has strong penetration ability, so UWB pulse locator can be used in complex electromagnetic environment such as indoor or basement while GPS locating system can only work within the visual range of GPS satellite. Secondly, UWB pulse locator can give both absolute location and relative location, while GPS provides absolute location. Thirdly, the locating accuracy of UWB pulse locator based on TDOA
technology can reach centimeter level, while GPS can only reach decimeter level. Finally, UWB pulse locator is cheaper and its cost-effective is higher than GPS [10].

### 2.2 TDOA Ranging Method

TDOA is a ranging method using the time difference of signal arrival. By measuring the transmission time of signal from source to sink, the distance between source and sink is calculated. Generally speaking, the wider the bandwidth of the signal, the narrower the pulse width, the higher the accuracy of time measurement, and the higher the accuracy of distance measurement. Therefore, this method is especially suitable for measurement of broadband low-power signals [11-13].

The TDOA ranging process is as follows [14].
(1) The anchor node sends a data packet to the node to be located and records the current time as $t_{1}$.
(2) The node to be located receives the data packet of the anchor node and returns an ACK.
(3) The anchor node receives the ACK of the node to be located and records the current time as $t_{2}$.
(4) The anchor node calculates the time difference $\Delta t=t_{2}-t_{1}$, and calculates the distance between the anchor node and the node to be located $d=C \times \Delta t / 2$, where C is light speed.

From the process of TDOA ranging, it can be seen that in order to measure the distance, the anchor node and the node to be located need to communicate twice. Therefore, this ranging method is also called two-way ranging.

## III. PRINCIPLE OF TWO-DIMENSIONAL LOCATION ALGORITHM

### 3.1 Analysis of Two-dimensional Location

In two-dimensional plane, two anchor nodes can not uniquely determine the location of the node to be located, and three anchor nodes are required for two-dimensional location at least. Next, we will discuss the problem of location with three anchor nodes.

Intuitively, the problem to locate $T$ with three anchor nodes $A_{1}, A_{2}$ and $A_{3}$ is just to seek the intersection of three circles. Suppose the coordinates of three anchor nodes are known, and the measured distances from three anchor nodes to T are $d_{1}, d_{2}$ and $d_{3}$ respectively. Draw three circles with $\mathrm{A}_{1}, \mathrm{~A}_{2}, \mathrm{~A}_{3}$ as the centers and with $d_{1}, d_{2}, d_{3}$ as the radius. If ranging is accurate, three circles will intersect at the same point. The coordinate of T can be obtained by seeking the intersection of three circles, as shown in Figure 1.


Figure 1. Three measured distances are equal to the actual distances
However, in practical application, ranging is not accurate, and the distance measured by the anchor node is not equal to the actual distance generally. Therefore, three circles do not intersect at the same point, as shown in Figure 2, 3 and 4, so T cannot be located by seeking the intersection of three circles.


Figure 2. Three measured distances are larger than the actual distances


Figure 3. Three measured distances are smaller than the actual distances


Figure 4. $d_{1}$ and $d_{3}$ are larger than the actual distances while $d_{2}$ is smaller than the actual distance

In order to solve locating problem generally, we discuss the problem of seeking the intersection of circles from the perspective of geometry.

### 3.2 Intersections of Two Circles

As shown in Figure 5, suppose that there are two intersecting circles in the plane, the center coordinates are $\mathrm{A}_{1}\left(x_{1}, y_{1}\right)$ and $\mathrm{A}_{2}\left(x_{2}, y_{2}\right)$, and the radii are $d_{1}$ and $d_{2}$ respectively. Next, we will seek the intersections of two circles.


Figure 5. Schematic diagram of seeking the intersections of two intersecting circles
Let the coordinate of the intersection is $(x, y)$, and get simultaneous equations

$$
\left\{\begin{array}{l}
\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}=d_{1}^{2}  \tag{1}\\
\left(x-x_{2}\right)^{2}+\left(y-y_{2}\right)^{2}=d_{2}^{2}
\end{array}\right.
$$

Subtract the first equation from the second one and get the binary linear equation

$$
\begin{equation*}
2\left(x_{1}-x_{2}\right) x+2\left(y_{1}-y_{2}\right) y=x_{1}^{2}-x_{2}^{2}+y_{1}^{2}-y_{2}^{2}+d_{2}^{2}-d_{1}^{2} \tag{2}
\end{equation*}
$$

It is easy to see that if point $(x, y)$ satisfies equations (1), $(x, y)$ must satisfy equation (2). On the
contrary, $(x, y)$ satisfies equation (2), but it may not satisfy equations (1).
From the viewpoint of geometric, binary linear equation (2) is the straight line MN through the intersections of two circles. Therefore, the intersection $(x, y)$ of two circles must be on the straight line MN. On the contrary, the point on the straight line MN is not necessarily the intersection of two circles. Thus, the process of transforming binary quadratic equations (1) to binary linear equation (2) is to convert the problem to seek the intersection $(x, y)$ of the two circles into the problem to seek the point $(x, y)$ on the straight line MN.

Generally speaking, as long as two circles are not concentric, whether they are separated or tangent, this transformation process is feasible, as shown in Figure 6. In particular, if two circles are tangent, the straight line MN is the common tangent of $\odot \mathrm{A}_{1}$ and $\odot \mathrm{A}_{2}$.



Figure 6. Schematic diagram of finding the intersections of two circles

### 3.3 Intersections of Three Circles

We can seek the intersections of three circles $\odot A_{1}, \odot A_{2}$ and $\odot A_{3}$ in following ways. At first, seek the intersections of two circles $\odot \mathrm{A}_{1}$ and $\odot \mathrm{A}_{2}$, secondly, seek the intersections of two circles $\odot \mathrm{A}_{1}$ and $\odot \mathrm{A}_{3}$, at last, seek the common points of two groups of intersections.

As shown in Figure 7, the problem to seek the intersections of $\odot A_{1}$ and $\odot A_{2}$ is converted into the problem to seek points on the straight line MN , and the problem to seek the intersection of $\odot \mathrm{A}_{1}$ and $\odot \mathrm{A}_{3}$ is converted into the problem to seek points on the straight line PQ. In this way, the problem to seek the intersections of three circles is converted into the problem to seek the intersections of the straight line MN and PQ.


Figure 7. Conversion of the problem to seek the intersections of three circles (I)
Similarly, the problem to seek the intersections of three circles can also be converted into the problem to seek the intersections of straight line MN and RS, as shown in Figure 8.


Figure 8. Conversion of the problem to seek the intersections of three circles (II)
The problem to seek the intersections of three circles can also be converted into the problem to seek the intersections of straight line PQ and RS, as shown in Figure 9.


Figure 9. Conversion of the problem to seek the intersections of three circles (III)

The analysis above shows that there are three methods to convert the problem to seek the intersections of three circles into the problem to seek the intersections of two straight lines. In this way, we can obtain three intersections theoretically. Then, are these three intersections the same point?

Theorem If $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and $\mathrm{A}_{3}$ are not on a straight line, the straight lines $\mathrm{MN}, \mathrm{PQ}$ and RS converted from $\odot A_{1}, \odot A_{2}$ and $\odot A_{3}$ intersect at the same point, as shown in Figure 10.


Figure 10. Intersection of three straight lines converted from three circles

Proof The slope of MN $k_{1}=-\frac{x_{2}-x_{1}}{y_{2}-y_{1}}$ according to equation (2), and the slope of $\mathrm{A}_{1} \mathrm{~A}_{2} \quad k_{2}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$.
Because $k_{1} k_{2}=-1$, MN perpendiculars to $\mathrm{A}_{1} \mathrm{~A}_{2}$.

Similarly, $P Q$ perpendiculars to $\mathrm{A}_{1} \mathrm{~A}_{3}$ and $R S$ perpendiculars to $\mathrm{A}_{2} \mathrm{~A}_{3}$.
Because $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and $\mathrm{A}_{3}$ are not on a straight line, the slopes of $\mathrm{A}_{1} \mathrm{~A}_{2}, \mathrm{~A}_{2} \mathrm{~A}_{3}$ and $\mathrm{A}_{3} \mathrm{~A}_{1}$ are not equal to each other, so the slopes of straight lines $\mathrm{MN}, \mathrm{PQ}$ and RS are not equal to each other, that is, the straight lines MN, PQ and RS intersect each other.

In order to prove that the straight lines $\mathrm{MN}, \mathrm{PQ}$ and RS intersect at the same point, it is only necessary to prove that the intersection of MN and PQ is the same as the intersection of MN and RS.

Let the equations of three circles $\odot A_{1}, \odot A_{2}$ and $\odot A_{3}$ be

$$
\begin{align*}
& \left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}=d_{1}^{2}  \tag{3}\\
& \left(x-x_{2}\right)^{2}+\left(y-y_{2}\right)^{2}=d_{2}^{2}  \tag{4}\\
& \left(x-x_{3}\right)^{2}+\left(y-y_{3}\right)^{2}=d_{3}^{2} \tag{5}
\end{align*}
$$

According to the method described in 3.2, equation (4) and equation (4) subtract equation (3) respectively to obtain the equations

$$
\left\{\begin{array}{l}
2\left(x_{1}-x_{2}\right) x+2\left(y_{1}-y_{2}\right) y=x_{1}^{2}-x_{2}^{2}+y_{1}^{2}-y_{2}^{2}+d_{2}^{2}-d_{1}^{2}  \tag{6}\\
2\left(x_{1}-x_{3}\right) x+2\left(y_{1}-y_{3}\right) y=x_{1}^{2}-x_{3}^{2}+y_{1}^{2}-y_{3}^{2}+d_{3}^{2}-d_{1}^{2}
\end{array}\right.
$$

The solution of equations (6) is the intersection coordinates of MN and PQ .

Similarly, equation (3) and equation (5) subtract equation (4) respectively to obtain the equations

$$
\left\{\begin{array}{l}
2\left(x_{2}-x_{1}\right) x+2\left(y_{2}-y_{1}\right) y=x_{2}^{2}-x_{1}^{2}+y_{2}^{2}-y_{1}^{2}+d_{1}^{2}-d_{2}^{2}  \tag{7}\\
2\left(x_{2}-x_{3}\right) x+2\left(y_{2}-y_{3}\right) y=x_{2}^{2}-x_{3}^{2}+y_{2}^{2}-y_{3}^{2}+d_{3}^{2}-d_{2}^{2}
\end{array}\right.
$$

The solution of equations (7) is the intersection coordinates of MN and RS.

Comparing equations (6) and (7), it can be seen that the first equation of them is the same. The second equation of equations (6) subtracts its first equation, and the difference is the second equation of equations (7). According to the same solution principle of equations, equations (6) and equations (7) have the same solution. Therefore, the intersection of MN and PQ is the same as the intersection of MN and RS. QED

The theorem shows that although there are three ways to convert the problem to seek the intersections of three circles into the problem to seek the intersection of two straight lines formally, three intersections obtained by three ways are actually the same point. This is the theoretical basis of two-dimensional location algorithm.

### 3.4 Two-dimensional Location Algorithm

Suppose that three anchor nodes $\mathrm{A}_{1}\left(x_{1}, y_{1}\right), \mathrm{A}_{2}\left(x_{2}, y_{2}\right)$ and $\mathrm{A}_{3}\left(x_{3}, y_{3}\right)$ are not on a straight line, and the distances from them to the node to be located T are $d_{1}, d_{2}$ and $d_{3}$ respectively, then the coordinate $(x, y)$ of T satisfies the following equations

$$
\left\{\begin{array}{l}
\left(x-x_{1}\right)^{2}+\left(y-y_{1}\right)^{2}=d_{1}^{2}  \tag{8}\\
\left(x-x_{2}\right)^{2}+\left(y-y_{2}\right)^{2}=d_{2}^{2} \\
\left(x-x_{3}\right)^{2}+\left(y-y_{3}\right)^{2}=d_{3}^{2}
\end{array}\right.
$$

The second equation and the third equation subtract the first equation respectively, and get the binary linear equations

$$
\left\{\begin{array}{l}
2\left(x_{1}-x_{2}\right) x+2\left(y_{1}-y_{2}\right) y=x_{1}^{2}-x_{2}^{2}+y_{1}^{2}-y_{2}^{2}+d_{2}^{2}-d_{1}^{2}  \tag{9}\\
2\left(x_{1}-x_{3}\right) x+2\left(y_{1}-y_{3}\right) y=x_{1}^{2}-x_{3}^{2}+y_{1}^{2}-y_{3}^{2}+d_{3}^{2}-d_{1}^{2}
\end{array}\right.
$$

Mark $A=\left[\begin{array}{ll}2\left(x_{1}-x_{2}\right) & 2\left(y_{1}-y_{2}\right) \\ 2\left(x_{1}-x_{3}\right) & 2\left(y_{1}-y_{3}\right)\end{array}\right], \quad B=\left[\begin{array}{l}x_{1}^{2}-x_{2}^{2}+y_{1}^{2}-y_{2}^{2}+d_{2}^{2}-d_{1}^{2} \\ x_{1}^{2}-x_{3}^{2}+y_{1}^{2}-y_{3}^{2}+d_{3}^{2}-d_{1}^{2}\end{array}\right], \quad X=\left[\begin{array}{l}x \\ y\end{array}\right]$, then equations (9) can be written
as

$$
\begin{equation*}
A X=B \tag{10}
\end{equation*}
$$

According to the theorem, matrix A is invertible. Solve equations (10) and get the coordinate of T as

$$
\begin{equation*}
X=A^{-1} B \tag{11}
\end{equation*}
$$

If ranging is accurate, as shown in Figure 1, three circles intersect at one point, that is, the equations (8) have a unique solution. According to the same solution principle of the equations, the binary quadratic equations (8) and the binary linear equations (9) have the same solution. Solve the equations (9) and obtain the coordinates of T. That is formula (11). In practical application, ranging is not accurate generally. The measured distances $d_{1}, d_{2}$ and $d_{3}$ from three anchor nodes to the node to be located $T$ may not be equal to the actual distance from three anchor nodes to T, as shown in Figure 2, 3 and 4. On this condition, although the coordinate $(x, y)$ can also be obtained by formula (11), it is not the real coordinate of T , there is locating error.

## IV. SIMULATION OF TWO-DIMENSIONAL LOCATION ALGORITHM

Let the coordinates of three anchor nodes be $\mathrm{A}_{1}(0,0), \mathrm{A}_{2}(400,0)$ and $\mathrm{A}_{3}(200,400)$ respectively, and the coordinate of the node to be located T be $(150,150)$. The actual distances from $\mathrm{A}_{1}, \mathrm{~A}_{2}$ and $\mathrm{A}_{3}$ to T are $212.132,291.5476$ and 254.951 respectively. The actual distance is added with Gaussian white noise to simulate three measured distances $d_{1}, d_{2}$ and $d_{3}$. The simulating result is shown in Figure 11. The coordinate of T is $(152.96,153.21)$ and locating error is 4.37 .


Figure 11. Simulating result of two-dimensional location algorithm

## V. FACTORS AFFECTING LOCATING RESULT

### 5.1 Distribution of Anchor Nodes

In actual locating process, we find that the distribution of anchor nodes affects locating accuracy. We illustrate this problem by simulation. Three anchor nodes are used to locate a node to be located, and two simulation environments are designed. In the first simulation environment, three anchor nodes form an acute triangle. In the second simulation environment, three anchor nodes form an obtuse triangle. In each simulation environment, implement 100 measurement calculations on MATLAB. The measured distance is added with Gaussian white noise artificially to simulate the random ranging error.

In the first simulation environment, the coordinates of nodes are $A_{1}(0,0), A_{2}(400,0), A_{3}(200,447)$ and $T(100,20), \Delta A_{1} A_{2} A_{3}$ is an acute triangle. The simulating result is shown in Figure 12.


Figure 12. Simulating result of two-dimensional location algorithm when $\Delta A_{1} A_{2} A_{3}$ is an acute triangle

In the second simulation environment, the coordinates of nodes are $\mathrm{A}_{1}(0,0), \mathrm{A}_{2}(400,0), \mathrm{A}_{3}(200,50)$ and $T(100,20), \Delta \mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{~A}_{3}$ is an obtuse triangle. The simulating result is shown in Figure 13.


Figure 13. Simulating result of two-dimensional location algorithm when $\Delta A_{1} A_{2} A_{3}$ is an obtuse triangle
In each simulation environment, 100 coordinates of T are averaged to obtain the locating coordinates of T. The simulating results of two-dimensional location algorithm under two distributions of anchor nodes are shown in TABLE I.

## TABLE I. Simulating results of two-dimensional location algorithm under two distributions of anchor

 nodes| distribution of anchor <br> nodes | locating coordinates of T | average error |
| :---: | :---: | :---: |
| acute triangle | $(100.15,20.07)$ | 7.42 |
| obtuse triangle | $100.25,16.91)$ | 25.38 |

We find the following phenomena from Figure 12, Figure 13 and TABLE I. If three anchor nodes form an acute triangle, the results obtained from multiple locating are concentrated near the real location of T. The locating accuracy can be effectively improved after averaging. On the contrary, if three anchor nodes form an obtuse triangle and tend to a straight line, the results of multiple locating are scattered and the error increases obviously. Therefore, the distribution of anchor nodes affects the locating accuracy indeed.

The simulation results note that in the location environment with multiple anchor nodes, three suitable anchor nodes should be selected from the anchor nodes, so that they form an acute triangle.

### 5.2 Relative Location of Nodes

In actual locating process, we also find that the relative location of nodes affects locating accuracy. We illustrate this problem by simulation. Three anchor nodes are used to locate a node to be located, and two simulation environments are designed. In the first simulation environment, T is inside the triangle formed
by three anchor nodes. In the second simulation environment, T is outside the triangle. In each simulation environment, implement 100 measurement calculations on MATLAB. The measured distance is added with Gaussian white noise artificially to simulate the random ranging error.

In the first simulation environment, the coordinates of nodes are $A_{1}(0,0), A_{2}(400,0), A_{3}(200,450)$ and $T(200,150), T$ is inside $\Delta \mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{~A}_{3}$. The simulating result is shown in Figure 14.


Figure 14. Simulating result of two-dimensional location algorithm when $T$ is inside $\Delta A_{1} A_{2} A_{3}$
In the second simulation environment, the coordinates of nodes are $\mathrm{A}_{1}(0,0), \mathrm{A}_{2}(400,0), \mathrm{A}_{3}(200,450)$ and $T(1000,1000)$, $T$ is outside $\Delta A_{1} A_{2} A_{3}$. The simulating result is shown in Figure 15.


Figure 15. Simulating result of two-dimensional location algorithm when $T$ is outside $\Delta A_{1} A_{2} A_{3}$
In each simulation environment, 100 coordinates of T are averaged to obtain the locating coordinates of T. The simulating results of two-dimensional location algorithm under two relative locations of nodes are shown in TABLE II.

# TABLE II. Simulating results of two-dimensional location algorithm under two relative locations of nodes 

| relative location of <br> nodes | locating coordinates of $\mathbf{T}$ | average error |
| :---: | :---: | :---: |
| T is inside $\Delta \mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{~A}_{3}$ | $(199.67,150.15)$ | 6.89 |
| T is outside $\Delta \mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{~A}_{3}$ | $(1004.05,1003.92)$ | 36.20 |

We find the following phenomena from Figure 14, Figure 15 and Table 2. If $T$ is inside $\Delta A_{1} A_{2} A_{3}$, the results obtained from multiple locating are concentrated near the real location of T . The locating accuracy can be effectively improved after averaging. On the contrary, if $T$ is outside $\Delta A_{1} A_{2} A_{3}$, the results of multiple locating are scattered and the error increases obviously. Therefore, the relative location of nodes affects the locating accuracy indeed.

The simulation results note that in the location environment with multiple anchor nodes, if the approximate location of the node to be located is known, three suitable anchor nodes should be selected from the anchor nodes, so that the node to be located is in the triangle formed by three anchor nodes.

### 5.3 Convex Quadrilateral Strategy of Two-dimensional Location

According to the simulation results gained in 5.1 and 5.2, we propose the following convex quadrilateral strategy of two-dimensional location for the problem of how to arrange anchor nodes and select appropriate anchor nodes from multiple anchor nodes.
(1) In order to increase the area coverage, four anchor nodes are considered to be arranged, and four anchor nodes should form a convex quadrilateral.
(2) For a node to be located, three anchor nodes can be selected from four anchor nodes for locating.
(3) In order to get better locating result, three anchor nodes should meet two conditions: the anchor nodes form a non obtuse triangle, and this triangle covers the node to be located.

In practical application, $n(n>4)$ anchor nodes can be considered to be arranged according to the size and the complexity of the locating area, as well as the requirements for locating accuracy. We can arrange anchor nodes and select appropriate anchor nodes from multiple anchor nodes referencing the convex quadrilateral strategy of two-dimensional location [15].

## VI. SUMMARIES

In this paper, we studied the principle of two-dimensional location algorithm, verified the correctness of the algorithm by simulation on MATLAB, and proposed the optimization schemes for the algorithm. The
simulating results showed that the algorithm can achieve fairly accurate locating. We analyzed the factors affecting locating result, and proposed the convex quadrilateral strategy. The work has important reference value for the practical application of location algorithm based on TDOA.

However, these research results only stay in the stage of simulation experiment. Only by applying these results can we achieve the purpose of research. Next, we will implement these algorithms in C language based on VC++ 6.0, and compile the C language program code in Visual Studio to generate .DLL files for the upper computer to call and execute. In addition, we will use the actual UWB pulse locator to carry out the locating test in the real environment, and put the theoretical research results into application.

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## REFERENCES

[1] Corral-De-Witt D, Carrera E V, Matamoros-Vargas J A, et al. From E-911 to NG-911: Overview and Challenges in Ecuador. IEEE Access, 2018, 6:42578-42591.
[2] Huangbao Liu, Tao Wang, Gang Peng. An Improved Centroid Location Algorithm. Microcomputer and Application, 2013, 32(11):73-75.
[3] L Doherty, KSJ Pister, LE Ghaoui. Convex position estimation in wireless sensor networks. Infocom Twentieth Joint Conference of the IEEE Computer \& Communications Societies. IEEE, 2001.
[4] Jia D, Li W, Wangt P, et al. An advanced distributed MDS-MAP localization algorithm with improved merging strategy. IEEE International Conference on Information \& Automation. IEEE, 2017.
[5] Yuri Assayag, Horácio Oliveira, Eduardo Souto, Raimundo Barreto and Richard Pazzi. Indoor Positioning System Using Dynamic Model Estimation. Sensors 2020, 20, 7003.
[6] Ghulam Bhatti. Machine Learning Based Localization in Large-Scale Wireless Sensor Networks. Sensors 2018, 18, 4179.
[7] David Gualda, María Carmen Pérez-Rubio, Jesús Ureña, Sergio Pérez-Bachiller, José Manuel Villadangos, Álvaro Hernández, Juan Jesús García and Ana Jiménez. LOCATE-US: Indoor Positioning for Mobile Devices Using Encoded Ultrasonic Signals, Inertial Sensors and Graph-Matching. Sensors 2021, 21, 1950.
[8] Jiuzhen Liang. Wireless Locating System. Beijing: Electronic Industry Press, 2013:139-146.
[9] Łukasz Rykała, Andrzej Typiak and Rafał Typiak. Research on Developing an Outdoor Location System Based on the Ultra-Wideband Technology. Sensors 2020, 20, 6171.
[10] Haibin Tong, Ning Xin, Xianli Su, Tengfeng Chen and Jingjing Wu. A Robust PDR/UWB Integrated Indoor Localization Approach for Pedestrians in Harsh Environments. Sensors 2020, 20, 193.
[11] Yiming Yu, Yuan Yao, Xuehu Cheng. TDOA Locating Technology and Its Practical Application. China Radio, 2013(11):57-58+70.
[12] Mohamed Khalaf-Allah. Particle Filtering for Three-Dimensional TDoA-Based Positioning Using Four Anchor Nodes. Sensors 2020, 20, 4516.
[13] Peng Wu, Shaojing Su, Zhen Zuo, Xiaojun Guo, Bei Sun and Xudong Wen. Time Difference of Arrival (TDoA) Localization Combining Weighted Least Squares and Firefly Algorithm. Sensors 2019, 19, 2554.
[14] V. Navrátil, J. Krška, F. Vejražka, et al. Chained wireless synchronization algorithm for UWB-TDOA positioning.

2018 IEEE/ION Position, Location and Navigation Symposium (PLANS). IEEE, 2018.
[15] Ruiqi Sun, Zhonghang He, Lu Gao, Jinliang Bai. Station Layout Optimization Genetic Algorithm for Four Stations TDOA Location. Journal of Physics: Conf. Series (2019) 0620081176.

