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The Exchange Rate of RMB Affects Economic Growth: an Open Ramsey Model based on the Regulation of External Debt Stock

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Abstract:

The paper introduces real exchange rate into Ramsey Model, and also gives some numerical simulation about the effect of real exchange rate on economic growth. The paper constructs net foreign debt stock control open Ramsey model based on not fully open economic conditions. In the net foreign debt stock control open Ramsey model we found that:

first, when $r>\rho$, that is, when foreign debt interest rate is higher than consumption discount factor, the currency's appreciation will make this country has less per capita effective capital on the balanced growth path, this is not conducive to a country's economic growth in the long term. Second, when $r<\rho$, that is, the foreign debt interest lower than consumption time discount factor, the currency's appreciation will make this country has more per capita effective capital on the balanced growth path.

***Keywords**: Real exchange rate, Net foreign debt stock control, Open Ramsey model*

I. INTRODUCTION

The issue of RMB exchange rate has been one of the focal points of debates and games in the academic, business and political circles and other social circles. A deep understanding of the RMB exchange rate issue is of great significance for putting forward macro policy proposals that are suitable for China's national conditions. The theoretical study can help deepen the understanding of the economic growth effect of the real exchange rate, help recognize the law of the exchange rate's effect on economic growth in practice, and help formulate correct macroeconomic policies.

Hooper and Kohlhagen (1978) [1]and Kenen and Rodriek (1986) [2] argued that exchange rate depreciation would be beneficial in increasing trade, but the significance of the coefficient was not high.

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Dornbusch (1987) [3], Engel (1999) [4], Devereux and Engel (1999) [5], and Obstfeld (1995) [6] studied the effects of exchange rate changes on the prices of imported and exported goods as well as economic growth based on the new open macroeconomics framework. Williamson (2003) [7] argued that a competitive exchange rate level would stimulate industrial products to enter the world market. Ramsey, in his article "Mathematical Principles of Saving", established a micro-based model and then used the model to solve the intertemporal optimal allocation of resources and optimal consumption for the purpose of the study.

II. MATERIALS AND METHODS

2.1 Materials and Methodology

In the Ramsey model research, the direct extension of the open economy research often brings anomalous conclusions, which economists tend to correct through international credit market constraints, preference parameter changes, limited horizons and investment adjustment costs. The Ramsey model in the open economy is an important extension of the traditional Ramsey model, which further extends the examination of a country's economic growth to the international context, facilitating the analysis of many international financial issues and providing important guidance for macroeconomic policy formulation.

2.1.1 Open Economy ramsey model based on international borrowing and lending constraints

The Ramsey model in an open economy allows the movement of goods between countries and international borrowing and lending. The model assumes that only one tangible good is included, and that foreigners can purchase domestic products while domestic residents can purchase foreign products. The ownership of domestic and foreign capital is assumed to be perfectly substitutable means of storing value and therefore to have the same rate of return r. The variable r is the unique world interest rate. The size of households is assumed to grow exogenously constant at rate n.

The model analyzes the behavior of residents, the behavior of manufacturers, and the open economy conditions respectively.

The behavior of representative residents is the result of optimal choice under the given constraints. Assuming that the size of residents' utility depends only on their consumption level, the utility function is defined as:

$$U=\int_{t=0}^{\infty}e^{-\rho t}u(c(t))L(t)dt \tag{1}$$

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c(t) is the per capita consumption, L(t) is the household size size, and ρ is the time preference rate. u(-) is the instantaneous utility function, which indicates the utility level of each member of the household at a given moment. The instantaneous utility function takes the following form:

$$u(c)=\frac{c^{(1-\theta)}}{(1-\theta)} \quad , \quad \theta>0 \tag{2}$$

The model uses assets a(t) to represent the net assets per inhabitant of the household. The residents of the household are competitive. The budget constraint for a representative resident is then:

$$\dot{a}=w+(r-n)a-c \tag{3}$$

Maximizing the utility function of the residents under the budget constraint (3) yields the Euler equation for consumption:

$$\frac{\dot{c}}{c}=\frac{r-\rho}{\theta} \tag{4}$$

To impose a constraint on the level of external debt, the model divides capital into two types: one that can be used as collateral for external debt, i.e. physical capital, and one that does not act as collateral for external debt, i.e. human capital. Now assume that the production function involves two types of capital:

$$\hat{y}=f(\hat{k},\hat{h})=A\hat{k}^{\alpha}\hat{h}^{\eta} \tag{5}$$

The Cobb-Douglas production function is used for the function. Where $\hat{k}$ is the physical capital per unit of effective labor, $\hat{h}$ is the human capital per unit of effective labor, α is the share of physical capital, η is the share of human capital, and $0<\alpha<1$, $0<\eta<1$, and $0<\alpha+\eta<1$.

Assume that the depreciation rate δ is the same for both types of capital. Since physical capital becomes collateral, then the net return $f_k-\delta$ on that capital is equal to the world interest rate r at all

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points in time, where f_k is the marginal output of capital. Thus, the expression for f_k determined by the Cobb-Douglas function implies that:

$$k = \alpha y / (r + \delta) \tag{6}$$

The above equation ensures that the ratio of physical capital to GDP, k/y, is stable throughout the shift to the steady state. This equation together with the production function gives a simplified expression for the production function:

$$\hat{y} = \tilde{A}\hat{h}^{\varepsilon} \tag{7}$$

Where, $\tilde{A} = A^{1/(1-\alpha)} \cdot [\alpha / (r+\delta)]^{\alpha/(1-\alpha)}$, and $\varepsilon = \eta / (1-\alpha)$.

Assume that the per capita assets of the country are a, the per capita physical capital is k, the per capita human capital is h, and the sum of per capita physical capital and per capita human capital is equal to m. Define d as the net liability of the home country to the foreigner, that is, the net equity that the foreigner has in the home country, then:

$$d = m - a = k + h - a \tag{8}$$

The optimality condition for the firm will still be that marginal output equals factor prices, and the budget constraint can be further written as:

$$\frac{d\hat{a}}{dt} = \frac{d\hat{k}}{dt} + \frac{d\hat{h}}{dt} + \frac{d\hat{d}}{dt} = a\hat{k}^{\alpha}\hat{h}^{\eta} - (r+\delta)\cdot(\hat{k} + \hat{h} - \hat{a}) - (g+n+\delta)\cdot\hat{a} - \hat{c} \tag{9}$$

Where letters with "^" above them all indicate the effective quantity of the indicator. From equations (6), (7) and (9) and the borrowing constraint d = k, a modified budget constraint can be obtained as follows:

$$\frac{d\hat{h}}{dt} = (1-\alpha)\cdot\tilde{A}\hat{h}^{\varepsilon} - (\delta + n + g)\cdot\hat{h} - \hat{c} \tag{10}$$

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Note that from equation (6), $\alpha \cdot \tilde{A}\hat{h}^{\varepsilon}$ corresponds to the rental expenditure on physical capital, and since d = k, the term corresponds to the net factor expenditure paid to foreigners and is therefore equal to the difference between GNP and GDP.

If it is assumed that residents produce goods directly, they will maximize their utility in equation (1) subject to the budget constraint in equation (3.10), and the optimality condition for consumption is:

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{(1-\alpha) \cdot \tilde{A}\varepsilon\hat{h}^{\varepsilon-1} - (\delta + \rho + \theta g)}{\theta} \tag{11}$$

Where, $(1-\alpha) \cdot \tilde{A}\varepsilon\hat{h}^{\varepsilon-1} = \tilde{A}\eta\hat{h}^{\varepsilon-1} = f_h$, and f_h is the marginal output of human capital. The equation can be further simplified as:

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{f_h - \delta - \rho - \theta g}{\theta} \tag{12}$$

Equations (10) and (12) together with the cross-sectional condition adequately describe the moving dynamics of the model and allow further analysis of the steady state of the economy.

2.1.2 Households and manufacturers in ramsey's model

The intertemporal equilibrium decision behavior of households and manufacturers embodies a solid micro-foundation and intertemporal analytical features for the analysis of macroeconomic problems. Together with the capital accumulation equation, it constitutes the main content of the model.

There are many households in a country, and their behavior mainly includes providing labor in exchange for wages, collecting interest income from assets, purchasing goods, and accumulating assets for saving. Ignoring population migration and the decision to simplify the birth rate of the population, the size of the household is assumed to grow exogenously constant at rate n. Assuming that the household size at time point 0 is normalized to 1, the household size at time point t is: $L(t) = e^{nt}$. Each household contains more than one working member in the current period, and these individuals with finite lifespans can make the model consider an infinitely continuing household through uninterrupted intergenerational transmission based on altruism. The household utility function is assumed to be:

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$$U = \int_{t=0}^{\infty} e^{-\rho t} u(c(t)) L(t) dt \tag{13}$$

C(t) is the total consumption level at time point t. $c(t) = C(t)/L(t)$ is the per capita consumption. ρ is the time preference rate. u(-) is the instantaneous utility function, which indicates the utility level of each member of the household at a given moment. For the convenience of the analysis, it is still assumed to be of the following functional form:

$$u(c) = \frac{c^{(1-\theta)}}{(1-\theta)} \tag{14}$$

Where $\theta > 0$, the utility function of this form is said to be a constant intertemporal elasticity of substitution utility function since the elasticity of substitution of this function is a constant 1/θ.

Denote the household's total net assets by A(t) and the household's net assets per capita by a(t), where a(t) is in real terms. The residents of the household are assumed to be competitive. Thus, the equation for the accumulation of household assets is:

$$\frac{dA}{dt} = r \cdot A + wL - C \tag{15}$$

Because a is the total per capita assets, then:

$$\dot{a} = \frac{d(A/L)}{dt} - na = \frac{1}{L} \cdot \frac{dA}{dt} - na \tag{16}$$

From equations (15) and (16), the equation for the level of assets per capita can be obtained:

$$\dot{a} = w + (r - n)a - c \tag{17}$$

If each household is allowed to borrow without limit at a given interest rate r(t), and residents borrow money for consumption and then borrow again to repay principal and interest, the household's debt continues to grow at the rate of interest r(t). To rule out the possibility of rolling debt issuance, researchers generally restrict the credit market by setting non-Ponzi game conditions. In this paper, we

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take the same approach and satisfy the following equation:

$$\lim_{t\to\infty}\left\{a(t)\cdot\exp\left[-\int_0^t\left[r(\lambda)-n\right]d\lambda\right]\right\}\geq 0 \tag{18}$$

In (18), a(t) denotes the net assets per capita of the household, while the net liabilities per capita can be expressed as - a(t). This condition implies that in the long run, the model can ensure that the growth rate of the debt level does not exceed r(t) by setting the growth rate of the household's per capita debt not to exceed the growth rate of r(t)-n.

There are many profit-maximizing manufacturers in a country. Manufacturers own capital and hire labor from competitive factor markets and sell their products in competitive markets to earn income. Although the manufacturer owns the capital stock rather than renting it from residents, the residents still have ownership of the manufacturer's net cash flows. Each vendor has the same production function, and at any given time, the firm has a certain amount of capital, labor, and knowledge. The production function is assumed to be: $Y(t)=F(K(t),A(t)L(t))$, where technological progress is considered to be labor-increasing, i.e., A is introduced into the function as a product with L. AL is called effective labor and is set to $\hat{L}$. The production function is scale payoff invariant with respect to the two independent variables, i.e., capital and effective labor. In mathematical expressions, this can be expressed as: $F(cK,cAL)=cF(K,AL)$. For all $c\geq 0$, the assumption of constant returns to scale can be rewritten the production function in an intensive form: $y=f(k)$, where $\mathrm{k}=\mathrm{K}/\mathrm{AL}$, $\mathrm{y}=\mathrm{Y}/\mathrm{AL}$, $\mathrm{f}(\mathrm{k})=\mathrm{f}(k,1)$, k denotes the amount of capital per unit of effective labor and y denotes the output per unit of effective labor.

$$\dot{K}=I \tag{19}$$

The firm is required to pay a wage rate w per unit of labor L. Thus, the manufacturer's profit at any point in time is determined by the following equation.

$$\pi=F(K,\hat{L})-\omega L-I$$

To analyze the present value of the manufacturer's profit, we define $\overline{r}(t)$ as the average interest rate from time 0 to time t. This rate can be determined by the following equation.

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$$\bar{r}(t) = \frac{1}{t} \cdot \int_0^t r(v)dv$$

Thus the present value of the manufacturer's profit can be further expressed as:

$$\pi = \int_0^\infty e^{-r(t)\cdot t} \cdot \left[F(K, \hat{L}) - \omega L - I \right] dt \tag{20}$$

The objective of the manufacturer is to maximize π by making choices of L and I at each point in time subject to the constraints of equation (19) and the initial value K(0). The optimization analysis is performed by constructing the Hamiltonian equation, which is given by:

$$H = e^{-r(t)\cdot t} \cdot \left\{ \left[F(K, \hat{L}) - \omega L - I \right] + qI \right\} \tag{21}$$

Where q is the shadow price of $\dot{K} = I$. Since the present value Hamiltonian equation is constructed, q is the value of the unit of capital measured in terms of products at time point t. That is, q represents the shadow price of the investment measured in terms of contemporaneous products. Therefore, the present value shadow price is: $v = q \cdot e^{-r(t)\cdot t}$

The first-order condition for maximizing the manufacturer's profit is:

$$\frac{\partial H}{\partial L} = 0 \Rightarrow \left[f(\hat{k}) - \hat{k} f'(\hat{k}) \right] . e^{gt} = w \tag{22}$$

$$\frac{\partial H}{\partial I} = 0 \Rightarrow q = 1 \tag{23}$$

$$\dot{v} = -\frac{\partial H}{\partial K} \Rightarrow \dot{q} = rq - f'(k) \tag{24}$$

At the meantime, the cross-sectionality condition is:

$$\lim_{t \to \infty} \left[v(t) K(t) \right] = 0 \tag{25}$$

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From (23) and (24), it follows that:

$$f'(k) = r \tag{26}$$

2.1.3 Construction of open ramsey model based on the regulation of external debt stock

For ordinary commodities, prices are the result of intersecting supply and demand curves, and only after the actual factors affecting supply and demand change, prices adjust with the changes in supply and demand. And the supply and demand response of an asset in the market is necessary to adjust the holding stock of this asset. In other words, the price of an asset changes because the market as a whole changes its assessment of the value of that asset, so it is possible for the price of an asset to change (even considerably) with little or no fundamental change in supply and demand. Considering the properties of the exchange rate as an asset price and the government's management objective based on stock regulation, the paper again develops an open Ramsey model based on stock regulation of foreign debt.

In an open economy, goods are allowed to move between countries and capital is allowed to move internationally, i.e., international borrowing is allowed. However, since the country is an incomplete open economy with non-free flows under the capital account, a country's government can impose limits on the size of borrowing according to the needs of economic development. Drawing on Hu, Cui, and Xu (2011)'s approach to constraining the size of external debt, the ratio of external debt size to capital is assumed to be $\mu(t)$, i.e.

$$\mu(\mathrm{t}) = D(t) / K(t) \tag{27}$$

Factors that affect changes in a country's external assets or liabilities are changes in the supply of various assets (e.g., domestic currency, domestic bonds, foreign bonds) and changes. It is further assumed that the country adjusts the change in the ratio of a country's total foreign assets or liabilities to capital $\mu(t)$ mainly by changing the exchange rate, and the real exchange rate is a policy variable that can be regulated by the government. Then μ is clearly a function of the real exchange rate q. Defining $\mu(q)$, $\mu(\mathrm{q}) > 0$ means having net foreign liabilities and $\mu(\mathrm{q}) < 0$ means having net foreign assets. From the economic significance of $\mu(q)$, it is clear that $\mu'(\mathrm{q}) < 0$. Thus equation (27) can be further written as:

$$\mu(\mathrm{q(t)}) = D(t) / K(t) \tag{28}$$

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According to equation (28), the additional external borrowing of this economy at time t is

$$\dot{D}(t)=\mu(q(t))\cdot\dot{K}(t) \tag{29}$$

The derivation of the capital accumulation equation is similar to that related to Section 2. Defining d as the net liability of the home country to the foreigner, that is, the net equity that the foreigner has in the home country, then

$$d=k-a \tag{30}$$

According to equations (17), (22), (26) and (30), we can obtain:

$$\dot{d}=\dot{k}-f(k)+c+rd \tag{31}$$

Using equations (29) and (30), (31) is organized as follows:

$$\left(1-\mu\left(q\right)\right)\dot{k}=f(k)-c-\mu\left(q\right)rk$$

This equation can be further simplified as:

$$\dot{k}=\frac{1}{\left(1-\mu\left(q\right)\right)}\left[f(k)-c-\mu\left(q\right)rk\right] \tag{32}$$

Thus, the capital dynamics equation based on the regulation of the external debt stock is obtained. For a given k, it means that the level of c at time $\dot{k}=0$ is determined by equation $f(k)-c-\mu\left(q\right)rk=0$. Equation (32) determines k and the dynamics of $y=f(k)$ over time. The change in c over time can be translated into consumer behavior based on the previous consumer constraint and the relationship between assets per capita and capital per capita by maximizing U in (13) subject to constraint (32). The following Hamiltonian function is still solved using the Hamiltonian function defined as follows:

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$$H = u[c] \cdot e^{-(\rho-n)t} + \lambda \frac{1}{(1-\mu(q))}\left[f(k) - c - \mu(q) rk \right] \tag{33}$$

$$\frac{\partial H}{\partial c} = 0 \Rightarrow \frac{\lambda}{(1-\mu(q))} = u'(c) e^{-(\rho-n)t} \tag{34}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial k} \Rightarrow \dot{\lambda} = -\frac{\lambda}{(1-\mu(q))}\left[f'(k) - \mu(q) r \right] \tag{35}$$

The cross-sectional conditions are as follows:

$$\lim_{t \to \infty}\left[\lambda(t) k(t) \right] = 0 \tag{36}$$

Further, the Euler equation for consumption can be obtained as:

$$\frac{\dot{c}}{c} = \frac{\left[f'(k) - \mu(q) r \right] - \rho(1-\mu(q))}{\left[1-\mu(q) \right]\theta} \tag{37}$$

2.2 Long-run Stability and Dynamic Analysis of the Model

The economy reaches the steady state when $\dot{k} = 0$ and $\dot{k} = 0$:

$$c^* = f(k^*) - \mu(q) rk^* \tag{38}$$

$$f'(k^*) = \rho + (r - \rho)\mu(q) \tag{39}$$

Equation (38) describes the conditions for market clearing to be satisfied in equilibrium in the commodity market; and equation (39) shows that the marginal productivity of the capital stock at equilibrium is related to the discount factor of consumption, the level of the real exchange rate, the

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elasticity of substitution, and the rate of technological progress. For the convenience of the analysis, the production function is still set to the Cobb-Douglas form, $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$, and the population growth rate and the technology growth rate are set to 0. Then, from Equations (38) and (39), the steady-state values of c and k can be obtained.

$$k^{*} = \left[\frac{\rho + (r-\rho)\mu(q)}{\alpha}\right]^{\frac{1}{\alpha-1}} \tag{40}$$

$$c^{*} = \left[\frac{\rho + (r-\rho)\mu(q)}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} - r\mu(q)\left[\frac{\rho + (r-\rho)\mu(q)}{\alpha}\right]^{\frac{1}{\alpha-1}} \tag{41}$$

Taking the derivative with respect to q for both sides of equation (40) yields:

$$\frac{dk^{*}}{dq} = \frac{1}{\alpha-1}\left[\frac{\rho + (r-\rho)\mu(q)}{\alpha}\right]^{\frac{1}{\alpha-1}-1} \cdot \frac{(r-\rho)\mu'(q)}{\alpha} \tag{42}$$

From (42) and $\mu'(\mathrm{q}) < 0$, it is clear that the equilibrium capital level will rise when $r > \rho$, $\frac{dk^{*}}{dq} < 0$, i.e., with the appreciation of the local currency exchange rate, and fall when $r < \rho$, $\frac{dk^{*}}{dq} > 0$, i.e., with the appreciation of the local currency exchange rate. This gives the relationship between equilibrium capital level and real exchange rate in the external debt stock model.

The nonlinear equations expressed in Equations (32) and (37) are replaced by linear approximation equations in the vicinity of the equilibrium growth path, and the saddle point path is further described for this purpose. Taking the first-order Taylor approximation for Equations (32) and (37) in the vicinity of $k = k^{*}$ and $c = c^{*}$:

$$\dot{c} = \frac{\partial \dot{c}}{\partial k}\left[k - k^{*}\right] + \frac{\partial \dot{c}}{\partial c}\left[c - c^{*}\right] \tag{43}$$

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$$\dot{k} = \frac{\partial \dot{k}}{\partial k}\left[k - k^*\right] + \frac{\partial \dot{k}}{\partial c}\left[c - c^*\right] \tag{44}$$

$\partial \dot{c} / \partial k$, $\partial \dot{c} / \partial c$, $\partial \dot{k} / \partial k$, $\partial \dot{k} / \partial c$ are taken when $k = k^*$ and $c = c^*$. Defining $\tilde{c} = c - c^*$ and $\tilde{k} = k - k^*$, it follows that:

$$\dot{\tilde{c}} = \frac{\partial \dot{c}}{\partial k}\tilde{k} + \frac{\partial \dot{c}}{\partial c}\tilde{c} \tag{45}$$

$$\dot{\tilde{k}} = \frac{\partial \dot{k}}{\partial k}\tilde{k} + \frac{\partial \dot{k}}{\partial c}\tilde{c} \tag{46}$$

According to Equations (37) and (45), it is obtained that:

$$\dot{\tilde{c}} = \frac{f''(k^*)c^*}{\left[1 - \mu(q)\right]\theta}\tilde{k} \tag{47}$$

Similarly, according to Equations (32) and (46), it is obtained that:

$$\dot{\tilde{k}} = \rho\tilde{k} - \tilde{c} \tag{48}$$

$$\dot{\tilde{\tilde{k}}} = \beta\tilde{\tilde{k}} - \tilde{\tilde{c}} \tag{49}$$

Dividing both sides of Equation (48) by $\tilde{c}$ and both sides of Equation (49) by $\tilde{k}$, the expressions for the growth rates of $\tilde{c}$ and $\tilde{k}$ are obtained as follows:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{f''(k^*)c^*}{\left[1 - \mu(q)\right]\theta}\frac{\tilde{k}}{\tilde{c}} \tag{50}$$

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$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \rho - \frac{\tilde{c}}{\tilde{k}} \tag{51}$$

Let $\tilde{c}$ and $\tilde{k}$ have the same growth rate and both are δ, then according to equations (50) and (51) we can get:

$$\frac{\dot{\tilde{c}}}{\tilde{k}} = \frac{f''(k^*)c^*}{[1-\mu(q)]\theta}\frac{1}{\delta} \tag{52}$$

Substituting Equation (52) into Equation (51), we get:

$$\delta = \rho - \frac{f''(k^*)c^*}{[1-\mu(q)]\theta}\frac{1}{\delta} \tag{53}$$

Rearranging equation (53) yields:

$$\delta^2 - \rho\delta + \frac{f''(k^*)c^*}{[1-\mu(q)]\theta} = 0 \tag{54}$$

Solving equation (54) for:

$$\delta = \frac{\rho \pm \left\{\rho^2 - 4f''(k^*)c^* / [1-\mu(q)]\theta\right\}^{\frac{1}{2}}}{2} \tag{55}$$

If δ is positive, then $\tilde{c}$ and $\tilde{k}$ are growing, which indicates that the economy is not following a straight line towards the equilibrium point (k^*, c^*), but is deviating from it along a straight line (k^*, c^*). Therefore, if the economy is converging to (k^*, c^*), then δ is negative and a check of the above equation shows that there is only one value of δ, i.e., $\frac{\left\{\rho^2 - 4f''(k^*)c^* / [1-\mu(q)]\theta\right\}^{\frac{1}{2}}}{2}$ is negative, set

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to δ_1. The Equation $\frac{\dot{\tilde{c}}}{\dot{\tilde{k}}} = \frac{f''(k^*)c^*}{[1-\mu(q)]\theta}\frac{1}{\delta 1}$ represents the relationship between $\tilde{c}$ and $\tilde{k}$ in the process towards equilibrium and also determines the saddle point path.

III. CONCLUSION

Once the saddle point path, the capital stock and consumption levels at equilibrium, and the production function are determined, the relationship between the real exchange rate and economic growth can be simulated numerically. In this paper, we examine the cases of $\mu(q)$ = -10%, -30%, 10%, and 30%, respectively. The results of the simulation are shown in Fig 1.

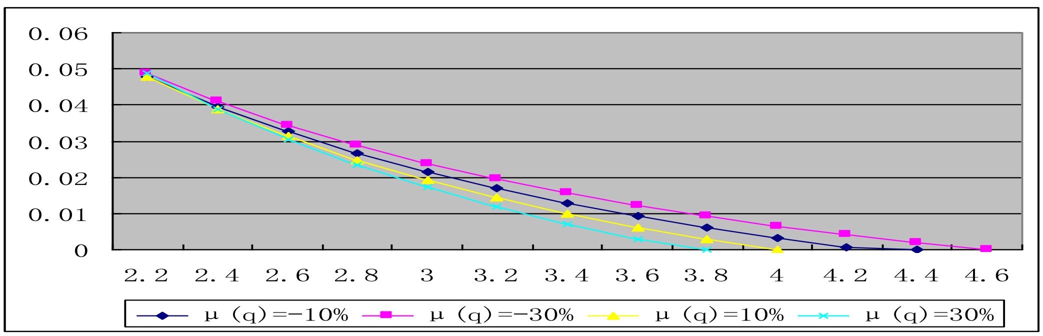

Fig 1: Numerical simulation of economic growth rate for different values of μ(q) (r=6%)

The relationship between the simulated $\mu(q)$ -value and economic growth is reversed when compared to the borrowing rate set at 7%. Fig 2 shows that the capital stock in equilibrium and the economic growth rate on the saddle point path will increase as $\mu(q)$ rises.

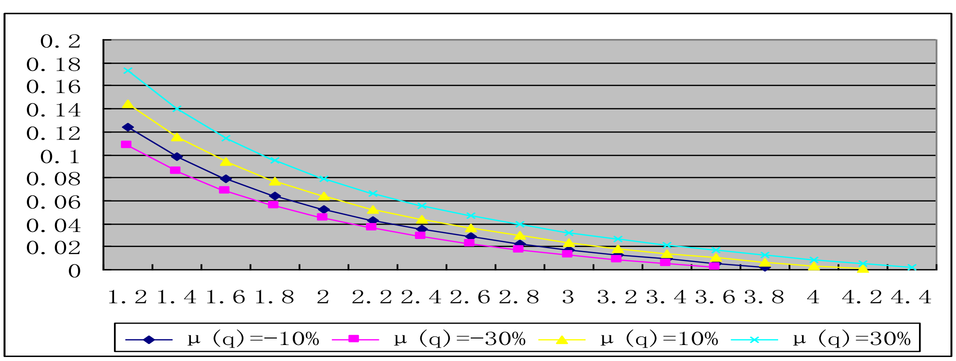

Fig 2: Numerical simulation of economic growth rate for different values of μ(q) (r=2%)

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Fig 1 and Fig 2 show that the relationship between a country's debt ratio and economic growth depends on the size of the interest rate at which a country borrows debt. Depending on the interest rate at which it borrows debt, the relationship between $\mu(q)$ and economic growth shows different relationships. An in-depth analysis of the parameters of the above graphs reveals that there is a monotonically decreasing relationship between $\mu(q)$ and economic growth when $r>\rho$, and a monotonically increasing relationship between $\mu(q)$ and economic growth when $r<\rho$. This relationship is similar to the one analyzed above between $\mu(q)$ and economic growth. In fact, this relationship illustrates that when all other conditions are equal, the higher the equilibrium capital level, the higher the rate of economic growth before reaching equilibrium.

Proposition 1: An appreciation of the local currency exchange rate brings about a decrease in the level of capital on the equilibrium growth path when other variables are constant and the interest rate on external liabilities is higher than the time discount factor for consumption; an appreciation of the local currency brings about an increase in the level of capital on the equilibrium growth path when the interest rate on external liabilities is lower than the time discount factor for consumption.

Proposition2: When the interest rate on external debt is higher than the time discount factor for consumption, the appreciation of the local currency exchange rate causes a decrease in the growth rate of a country's economy; when the interest rate on external debt is lower than the time discount factor for consumption, the appreciation of the local currency exchange rate causes an increase in the growth rate of a country's economy.

The stock model of external debt emphasizes the influence of exchange rate on a country's external liabilities or external assets, which reflects the property of exchange rate as asset price and reveals the intrinsic mechanism of exchange rate as a policy variable affecting economic growth. This largely reflects the basic national conditions of China and is applicable to the period when a large amount of foreign assets or liabilities are accumulated in the process of economic development.

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