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Recycling Decision Analysis of Power Battery Third-party Recyclers Considering Government Rewards and Punishments

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Abstract:

The booming development of new energy vehicles has brought a continuous increase in the de-mand for power batteries and the amount of scrap. A closed-loop supply chain consisting of one manufacturer, one downstream seller and one third-party recycler is constructed. Then the influ-ence of the government rewards and punishments policy and the proportion of responsibility sharing to achieve the coordination of the closed-loop supply chain are studied. The results show that: (1) When the government does not interfere in the recycling of the closed-loop supply chain, the total market profit of power battery under the decentralized decision is always lower than that under the centralized decision. (2) The change of rewards and punishments ratio under the recy-cling channel of the third-party recyclers does not affect the retail price in the consumer market. (3) Under the government's policy, the market recovery rate is lowest and the wholesale price is highest when the downstream seller shares the recycling responsibility with the manufacturer. (4) Under certain conditions, the intervention of the government rewards and punishments mechanism will improve the overall profit of the closed-loop supply chain system and the market recovery rate, and finally realize the Pareto improvement of the power battery closed-loop supply chain.

Keywords: *Power battery, Closed-loop supply chain, Third-party recycling, Government rewards and punishments, Liability share.*

I. INTRODUCTION

New energy vehicle industry has been booming in recent years. Based on the data from EV Sales, the global sales of new energy vehicles reached 3.124 million in 2020, with a year-on-year increase of 41%. And It is predicted that the plug-in electric vehicle stocks may reach 11.70 million by the end of 2023[1]. While the number of installed power batteries is growing

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rapidly, a large number of power batteries have been retired. China Merchants Securities predicted that by 2025, the volume of retired power battery in China alone will be close to 134.49 GWh, corresponding to a market size around 35.4 billion RMB. Due to high performance requirements for power battery products of electric vehicles, the service cycle of the battery only accounts less than 40% of the whole life cycle of the electric vehicle. And more than 60% of the energy of the retired battery can still be utilized. Directly dispose of these batteries will lead to serious waste of resources and environmental pollution. What's more, the raw materials needed for power battery production, such as lithium, nickel and cobalt, are relatively scarce. Therefore, effective recycling of the power battery is not only important to protect the natural environment and promote the sustainable development of society, but also helpful to alleviate the high economic cost.

Although the electric vehicle market has been developing very well, the power battery recycling technology is still improving. No mature standardized large-scale recy-cling method exists yet. And the uniform third-party power battery recycling mechanism has not been built. Due to the unclear about benefits and costs of power battery recovery market, most power battery recyclers and market consumers also lack enough enthusiasm to participate in power battery recycling and utilization. However, the recycling of waste power battery products is not only the end of automobile power products, but also the source of the corresponding industrial chain. The strong effect on environmental pollution, cost saving and industrial development draws significant attention from the government. The policies to effectively stimulate the recycling of waste products in the power battery market need to be explored.

Third-party recycling enterprises provide a series of solutions for the reverse logistics market from product assembly, storage and transportation to terminal processing and regulatory documents, so as to help power battery manufacturers to realize the ex-tension of producer-related responsibilities. Call 2 Recycle has more than 30,000 recycling outlets in China and other countries, including wholesalers, distributors, public institutions and most of the participants. Company Tianqi Lithium, through cooperation with Gantai Energy Renewable Technology Co., Ltd., successfully separates the responsibility of production and recycling by entrusting the recycling to battery regeneration enter-prises. In the closed-loop supply chain, the manufacturer delivers the related recycling business to the third-party recycling enterprise for management, which can not only undertake the basic investment related to the recycling channel, but also make use of the professional technology of the recycling enterprise to effectively manage the recycling activities of used power batteries. The biggest advantage of this recycling channel lies in the rapid layout, large quantity and complete categories. Thus, the difficulty of power battery recovery in new energy vehicles is greatly reduced. At the same time, third-party recycling can also improve the recovery rate and recovery effect of waste

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products. The relatively diversified systematic solutions save the cost of layout in the upper, middle and downstream links for the members of the closed-loop supply chain, while the project-based model can specifically address the recycling needs of power battery manufacturers.

Therefore, we focus on the power battery recycling supply chain under the third-party recycler channel considering the government policy of rewards or punishments in this paper. Using the theory of Stackelberg model, the optimal decisions of manufacturers, sellers and recyclers are investigated. Based on the analyses about the influence from the government's policy, a decision-making and coordination mechanism of closed-loop supply chain based on government rewards and punishments is then proposed.

Third-party recycling mode is one of the most common and mature recycling modes. In this mode, the seller buys power battery products from the manufacturer and sells them to the consumer market, and then the recycler takes back the waste products through professional recycling channels. Third-party recycling channel shows the ad-vantages of wide cover range, high speed and high degree of specialization, but also exist the problem of no unified recycling standards and lack of third-party recycler qualifications. So the regulation from the government is necessary in order to reduce the loss in the closed-loop supply chain system and improve development of the market and the protection of environment.

Aiming at the recycling problem of third-party recyclers, we discuss the relevant literature from the following three parts. We will first focus on the influencing factors of recycling of waste products in the closed-loop supply chain. And then summarize the relevant recycling methods according to the recycling characteristics of used power batteries. After this, the research on recycling channels of waste products in the closed-loop supply chain is introduced. Based on the review, the research content of this paper is presented.

Many literatures have studied the factors affecting the recycling of waste products in the closed-loop supply chain market, including government policies and regulations, consumer preferences and attitudes, market economy, social environment and ecology. The Extended Producer Responsibility (EPR) defines that producers must take responsibility for the recycling, regeneration and disposal of products. In other words, producers' responsibility should be extended to the whole life cycle of products. Li and Zhu[2] studied the problem of recycling waste products by exchanging old for new under the centralized decision of manufactures with EPR constraints. They found the government's optimal policy's impact on the sales volume, total profit and waste product recycling volume depends on the cost structure, consumer structure, consumer utility and waste product recycling capacity of the manufacture.

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Zheng and Tian[3] studied the relation-ship between the government and relevant manufacturers from the perspective of EPR. From the analyses of a two-stage complete information dynamic game model between the government and manufacturers, they found that the government could ensure the restraint and incentive effect of EPR in order to promote production enterprises to recycle EOL products. Choi and Rhee[4] based on power battery products recycling measures and recycling technology in different countries around the world, proposed new solutions about the management of the electric car battery under the producer responsibility system, to ensure the economic benefits of power battery recycling and environmental sustainability.

In addition to the extended responsibility of the producer, the government's policy of rewards and punishments also plays an important role in the product recovery supply chain. Hong and Ke[5] took the government as the decision leader and each participant in the closed-loop supply chain as the decision followers, and studied the effect of the government and the manufacturer jointly subsidize the third-party recyclers. They found that the closed-loop supply chain system under the equilibrium state had the highest profit. Wang and Da[6] studied how participants in a closed-loop supply chain based on government incentives and punishments make relevant decisions on the recycling rate and quantity of waste products. With game theory analysis method, Ma et al[7]. studied the closed-loop supply chain of electric and electronic products in mixed channel sales and recycling, established the CLSC decision-making model, and analyzed the optimal parameters of government regulation and the optimal decision-making, benefits and so-cial welfare issues of participants in the closed-loop supply chain. Wang et al.[8] established a closed-loop supply chain decision-making model under decentralized and centralized decision-making based on the cost-sharing and profit-sharing contract model, and analyzed the decision-making behaviors and overall benefits of participants under different sharing ratios. Shi et al.[9] constructed cooperative and non-cooperative strategies by studying the impact of the reward and punishment mechanism in the recycling model of third-party recyclers on the manufacturer's strategy in the mixed closed-loop supply chain, and found that manufacturers, sellers and recyclers have cooperation motivations, but are affected by the government's incentive and punishment measures.

From the perspective of market, consumer preferences, economic and ecological factors will also affect product recycling. Sidiquea et al.[10] established an econometric model and analyzed the statistical data, and found that the income and consumption capacity of consumers in the closed-loop supply chain market, the recycling level of the recycling subject and demographic characteristics could all affect the recycling situation of waste products by the recycling party. Cao et al.[11], based on consumer preferences, studied the dual-recycling channel closed-loop supply chain model, which included manufacturers and sellers, and obtained the pricing decision-making results of each par-ticipant in the closed-loop supply

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chain. Natkunarajah et al.[12] built a recycling model of power battery closed-loop supply chain market from the perspectives of social eco-logical environment, basic economic conditions and recycling technology, and analyzed the recovery rate under different influencing factors. Liu and Gong[13] studied the recy-cling model in which vendors participated in the construction of recycling system, and analyzed the influence factors of each participant in the closed-loop supply chain and the results determined by different influence factors. From the influence factors of closed-loop supply chain, it can be found that there are few research on the government incentive policies of rewards and punishments, which play an important role in further clarifying corporate responsibility.

However, the recycling of waste power battery is different from that of general products. Some scholars have carried out a series of research on the recycling method from the perspective of product life cycle theory and system dynamics theory. The ap-plication of product life cycle theory and the life cycle assessment of waste products are important directions for scholars to study the recycling of battery products in the closed-loop supply chain. Ahmadi et al.[14], based on the product life cycle model, studied the discarded battery products from raw materials into the closed-loop supply chain to the final resale process, and found their usefulness in the actual environment. Heymans et al.[15] simulated the cost structure of household and residential energy and regulation, and studied the problem of resource conservation in waste product recycling to reduce energy rates and auxiliary costs. Zackrisson et al.[16] studied the optimization design of lithium-ion battery for plug-in hybrid vehicles based on LCA theory, and found that the extension of product life cycle can improve the utilization of resources and im-prove the level of ecological environment.

System dynamics theory is helpful to analyze the feedback characteristics of internal components of supply chain, so as to find the root of problems. Dyson et al.[17], in order to address the impact of solid waste generation on social sustainable development, established a set of finite sample models to predict the generation of equation waste, and found that this model could effectively avoid the drawbacks of least square regression and track the level of waste discharge. Kannan et al.[18] used GA model to solve the mixed integer linear programming model, and built a closed-loop supply chain network model of product return with multi-step, multi-cycle and multi-product. Comparing with the solution obtained by GAMS, they showed the best method of waste lead-acid battery material recovery. The related theories of game theory are also applied to the field of power battery recovery. Among them, You et al.[19], aiming at the waste battery pollution problem caused by the popularity of new energy vehicles, built cost models under different preconditions by using game theory and principal-agent theory, and then studied the quality of unilateral enterprises and the recycling benefits of waste products un-der related product recycling. Han et al.[20], based on the

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optimization theory of battery energy storage system, constructed a cost-benefit model under multiple agents and ap-plied the Nash equilibrium solution obtained by genetic algorithm to the planning and design of power battery energy storage stations. By constructing the optimization model of reverse supply chain network, Frank et al.[21] studied the recycling channel of waste power battery and found that by improving and optimizing the existing recycling structure, almost all of waste battery products could be recycled. Gong et al.[22] used queuing theory to simulate and analyze the matching degree of electric vehicles and batteries in the power battery closed-loop supply chain from the perspective of simulation, and studied the factors of the battery life cycle, production speed, battery elimination frequency and elimination rate of electric vehicles. Chuang et al.[23], based on the paperboy theory, analyzed the demand uncertainty closed-loop supply chain model for high-tech products, and studied multiple reverse logistics methods and their effects for manufacturers to recycle waste products and reproduce them. Lu et al.[24] considered the dual risks of market demand and recycling channels in the recycling and utilization of electric vehicle parts, and analyzed the impact of participants on the recycling price by establishing a two-stage closed-loop supply chain pricing model. Most of the current recycling systems are constructed from the perspective of manufacturers, and few of them are on the third-party recycling and the sharing of the recycling responsibility of the participants.

Classical recycling of waste products in closed-loop supply chain mainly includes three channels: manufacturer recycling, seller recycling and third-party recycling. Savaskan et al.[25] first proposed the problems related to recycling channels of closed-loop supply chain. They mainly analyzed three recycling channels for waste products: recycling by vendors, direct recycling by manufacturers and recycling by third-party recyclers. Both advantages and disadvantages of relevant recycling channels are discussed. Savaskan et al.[26] used game theory to analyze the influence of pricing of new products produced by manufacturers on the selection of recycling mode of waste products. Atasu et al.[27] discussed the recycling cost structure to another manufacturer of reverse channel optimal selection and found that the best reverse channel selection is made up of cost structure, the adjustment ability of manufacturers, the retailer's sales and recycled amount. Hong et al.[28] studied the recycling strategy of waste products under reverse logistics based on the electronic product recycling market and analyzed the re-tailer-led recycling method of waste electronic products. Based on the game theory, Yi et al.[29] classified three channels of manufacturer-led recycling, marker-led recycling and recycling without market dominant player, and found that the situation of no market leader has the highest profit for the closed-loop supply chain system. The subsequent research of the team[30] built a recycling model of waste products based on the recycling channels of recyclers, and analyzed the sales price, recycling price and recovery rate of products in third-party recycling channels under government incentives and punish-ments. Ma et al.[31] studied the

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factors of consumer subsidies in the dual-channel closed-loop supply chain and summarized the decision-making behaviors of different participants before and after the government implemented subsidy policies. The impact of consumer subsidies from the perspectives of consumers, the scale of closed-loop sup-ply chain and enterprises are also considered. Hong et al.[32], based on Stackelberg's game theory, constructed a product recycling pricing model of closed-loop supply chain under decentralized and centralized decisions. Wei et al.[33] built a waste recycling reengineering model based on different closed-loop supply chain recycling channels, and finally compared and analyzed the optimal decisions of participants and the total profits of the system under different closed-loop supply chain channels. Gao et al.[34], based on game theory, studied the supply-chain model dominated by manufacturers and sellers, and compared and analyzed the effects of environmental protection, sellers' participation, decision-making behavior of the main body and overall profit. Yan et al.[35] summarized the two sales channels of the remanufactured product, analyzed the decision-making behaviors of participants in the product closed-loop supply chain under different channels, and found that the environmental benefits of the manufacturer's recycling of waste products were better, but the manufacturer's own initiative was lost. Prakash and Barua[36] summarized the strategic selection of vendors and third-party recyclers in the product supply chain led by manufacturers through the analysis of the fuzzy analytic hierarchy process. Guarnieri et al.[37] proposed an analysis model of strategic choice based on cognitive mapping technology. Based on the views of decision makers, they conducted correlation analysis and summarized four types of actions that need to be executed: strategic, environmental, economic and social. Finally, the problem structure method is used to analyze and optimize the problem of the reverse logistics of electronic waste. Gao et al.[38] studied the influence of different channel power structures on the optimal decision and profit of the actors in the closed-loop supply chain related to product sales price and recycling effort in consideration of recycling efficiency and sales effort under expansion demand. Angelo et al.[39], based on the closed-loop supply chain decision problem, studied a set of effective optimal decision models to analyze the return and recovery of multi-product dynamic batch problems in the closed-loop supply chain. Based on government consumption subsidy scheme and dual-channel closed-loop sup-ply chain model, Ma et al.[40] studied the influence of consumption subsidy on du-al-channel closed-loop supply chain, and analyzed the decision-making effect of channel members before and after the implementation of government subsidy scheme. Zhen et al.[41] studied the extent to which the manufacturer's optimal alliance was affected by the recycling channel competition by establishing four recycling alliances: non-alliance, manufacture-seller, manufacture-recycler, manufacture-seller and recycler, and obtained the optimal decision of the participants under different recycling channels. Huang et al.[42] built four recycling channel models by studying the closed-loop supply chain system composed of manufacturers, distributors and recyclers, and found that the combined channel of retailers and third-party recyclers would have the highest recycling rate

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and maximum social benefits. Lin[43] established a closed-loop supply chain model in which different collection subjects participated, and analyzed the optimal selection of each participant under centralized and decentralized decision-making. In addition, the online recycling channel can effectively supplement the existing waste product recycling channel by combining online and offline methods. Zhang et al.[44] designed a recycling method of waste products composed of manufacturers, distributors and Internet recycling enterprises for the price difference between new products and remanufactured products, as well as the two channels of waste recycling, and studied the pricing and co-ordination strategy of the closed-loop supply chain system. Fallah et al.[45] studied the influence of simultaneous competition between two closed-loop supply chains and Stackelberg competition on the system profit, demand and revenue of the closed-loop supply chain, and obtained the optimal decision of the participants of the closed-loop supply chain and the optimal profit of the system by game theory. The research of power battery closed-loop supply chain decision mainly focuses on the decision choice of the participants, and seldom involves the research of the optimal decision under different proportions of responsibility sharing.

Based on relevant theories, the above literatures discussed the optimization problem of recycling network of closed-loop supply chain through relevant coordination mechanism and algorithm. However, there still exist some shortcomings. (1) Previous studies on government intervention in decision-making behavior of closed-loop supply chain mainly focus on subsidy measures, and there are few studies on government punishment measures. (2) For the research on the decision-making of power battery loop supply chain, the relevant research content mainly focuses on the decision-making and selection of supply chain members. The research on the decision-making and selection of participants based on the total profit of the closed-loop supply chain system is insufficient. (3) From the perspective of the manufacturer's recycling responsibility system, the existing system only focuses on the independent responsibility of the manufacturer for recycling, and there are not many studies on the sharing of the recycling responsibility of the participants. Therefore, in view of the above problems, this paper adopts the related theories and methods of closed-loop supply chain to analyze the differences between the benefit methods of the participating members under the recycling channel of the third-party power battery recyclers. This paper also explores the optimal decision results of the manufacturers and sellers of power battery in the closed-loop supply chain with or without recycling responsibility sharing and with different proportions of responsibility sharing under government subsidies and penalties. Meanwhile, the influence of government reward and punishment intensity on the profit of different participants is analyzed and possible solutions achieving the Pareto optimization of power battery closed-loop supply chain system are discussed.

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II. THE MODEL AND BENCHMARK CASE

We focus on closed-loop supply chain where third-party recycler plays a role in re-cycling enterprise's profit. Based on the decision-making and coordination of all parties in the supply chain under the recycling channel of third-party recyclers, this section focuses on the optimal decision-making of power battery recycling in the closed-loop sup-ply chain under government subsidies and penalties, so as to realize the Pareto optimization of power battery closed-loop supply chain system.

2.1 Problem Statement and Assumptions

Assume that there exists a closed-loop supply chain system composed by an auto-mobile power battery manufacturer, a downstream seller, a third-party recycler and consumers. Under the third-party recycling channel, the power battery manufacturer first wholesales products at a certain price to the seller as the leader of Stackelberg. Then the downstream seller sets the product price and sells them to consumers. When the power batteries are used by consumers to a certain extent, they will become waste products. At this time, the third-party recycler uses its own recycling network to recycle the waste power batteries at a certain price. In the end, the manufacturer buys back used power batteries from the recycler for remanufacturing. Let represent the power battery manufacturer, the downstream seller and the third-party recycler, respectively. And, the recycling decision process and related parameters are shown in Figure 1.

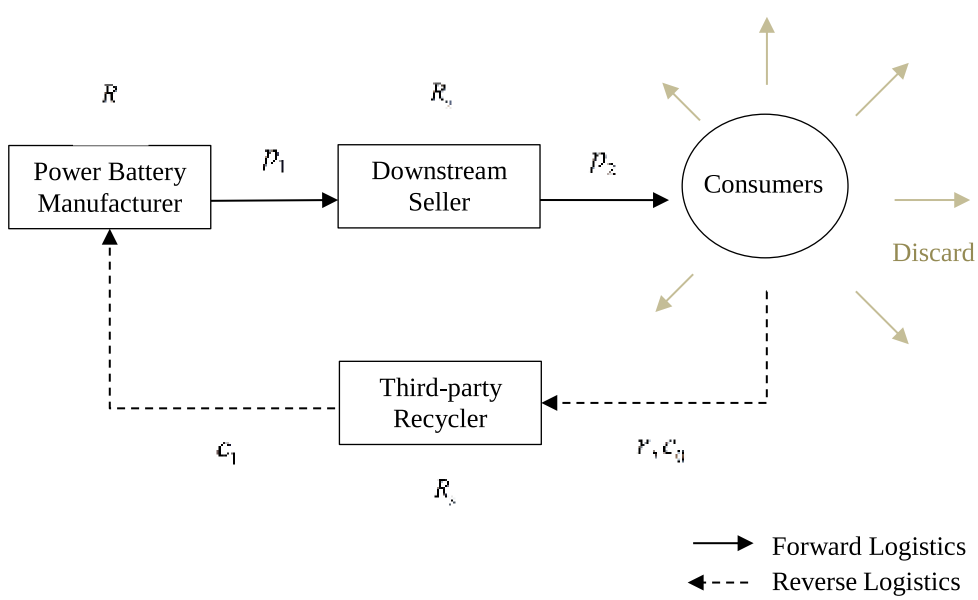

Fig 1: Recycling decisions under the third-party recycling channel

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Based on the practice and literature, the following assumptions are proposed in this section.

Assumption 1. p_1 represents the wholesale price of power battery products sold by the manufacturer, and p_2 represents the unit price sold by the seller to consumers. c_0, r, c_1 respectively represent the recycling price of the waste power battery products by the third-party recyclers, the recovery rate in the consumer market and the manufacturer's buyback price. In order to make the model meaningful, the price of power battery products sold by the seller should be greater than the price purchased from the manufacturer, and the cost of recycling waste products from consumers should be less than the cost of repurchasing waste products from the recycler, that is $p_2 > p_1$, $c_1 > c_0$.

Assumption 2. Power battery market demand is $D(p_2) = d - p_2$.

There is no difference in demand preference between remanufactured products and new products in the market. The market demand for this product is a monotonous decreasing function of sales price, without considering the uncertainty of market demand. d represents the total market demand when the selling price is zero, that is, the basic market size of the market. Let, $p_2 = e + p_1$, e ($e > 0$) refer to the amount that the seller raises the price of his products in order to make a profit. So the demand for the power battery manufacturer is $D(p_2) = d - e - p_1$.

Assumption 3. Let c_n, c_r represent the unit cost of manufacturing battery products with new materials and the unit cost of reconstructing new products with used batteries, respectively. The two products are homogeneous. Let $v = c_n - c_r$ and $v > c_0$, which is the difference between the unit cost of remanufacturing waste products and that of production with new materials. The positive reflects the cost saving advantage brought by remanufacturing. Therefore, manufacturers and recyclers become active in the remanufacturing activities of power battery recycling, and the industry alliance of manufacturers entrusting distributors to recycle waste products is established.

Assumption 4. Assume $I = wr^2/2$, where I is the investment needed to recover used power battery products; $w > 0$ is the difficulty of recycling used power battery products; and r is the recovery rate of used power battery products on the market.

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2.2 The Model

Based on the above assumptions, a closed-loop supply chain recycling model of third-party recyclers is built. With the sales price and recovery price of enterprises as the independent variable and the profit of enterprises with different participants as the dependent variable, the optimal recovery price and maximum profit under the third-party recycling channel are analyzed and discussed.

In the manufacture-led closed-loop supply chain, the manufacturer and the downstream seller aim to maximize their own interests. Based on the Stackelberg model, the power battery manufacturer, as the market leader, determines the new product price and the recycling price of the waste product from the third-party recycler. The downstream seller of the supply chain refers to the manufacturer's decision and determines the price of new products facing consumers, in order to maximize his or her own profit.

The manufacturer's profit includes the profit from the sales of new products and the surplus value from the recycling of used power batteries. That is

$$\pi_1 = (p_1 - c_n)D + vrD - c_1 rD \tag{1}$$

where D denotes the market demand of power batteries, $(p_1 - c_n)D$ represents the profit gained by sales to the downstream seller, vrD is the cost saving advantage, and $c_1 rD$ represents the cost of repurchasing the used products and remanufacturing.

The seller's profit is the difference in price from selling to the consumer market. That is the profit gained by the seller by buying power battery products from the manufacturer and selling them to consumers, which can be represented by

$$\pi_2 = (p_2 - p_1)D \tag{2}$$

The third-party recycler benefits from the sales of used power batteries, whose profit can be denoted as

$$\pi_3 = (c_1 - c_0)rD - wr^2 / 2 \tag{3}$$

where $(c_1 - c_0)rD$ represents the profit generated by the recycler by selling the recycled used power batteries to the manufacturer, and $wr^2 / 2$ represents the investment needed to recover the used power battery products.

Note that the profit of all the participants are increasing in the market demand.

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2.3 Benchmark Case: No Government Rewards and Punishments

2.3.1 Decentralized decisions

All enterprises will only consider the maximization of their own interests. Therefore, when enterprises involved in the closed-loop supply chain make decentralized decisions, the power battery manufacturer, the downstream seller and the third-party recycler can only participate in the recycling decisions of the whole closed-loop supply chain under the premise of maximizing their own profits.

For the power battery manufacturer, substitute $D(p_1)=d-e-p_1$ into Equation (1). Then we can get the expected profit function of the power battery manufacturer with respect to variables p_1, c_1, which is

$$\begin{aligned}\pi_{R1}&=(p_1-c_n)D+vrD-c_1rD\\&=(p_1-c_n)(d-p_2)+vr(d-p_2)-c_1r(d-p_2)\\&=\left[p_1-c_n+(v-c_1)r\right](d-p_2)\end{aligned}\tag{4}$$

For the downstream seller, since $D(p_2)=d-p_2$, the expected profit function of the seller with respect to variable p_2 can be obtained by putting it into Formula (2):

$$\pi_{R2}=(p_2-p_1)D=(p_2-p_1)(d-p_2)\tag{5}$$

For the third-party recycler, the expected profit function with respect to recovery r can be obtained by putting $D(p_2)=d-p_2$ into Equation (3):

$$\begin{aligned}\pi_{R3}&=(c_1-c_0)rD-wr^2/2\\&=(c_1-c_0)r(d-p_2)-wr^2/2\end{aligned}\tag{6}$$

The optimal decisions can be obtained by maximizing the expected profit function of power battery manufacturer and the downstream seller.

Theorem 1. Suppose there is a power battery market under the recovery of a third-party recycler, including three game players: one manufacturer, one downstream seller and one third-party recycler. p_{R1} and c_{R1} respectively represent the unit price of new products sold by the

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power battery manufacturer and the unit cost of purchasing used power batteries from the recycler under the third-party recycling channel. p_{R2} represents the unit price of new products sold by the downstream seller to consumers, and r_R represents the recovery rate of used power batteries. Then the optimal decisions are as follows: $p^*_{R1}=\frac{(d+c_n)(c_0-v)^2-2w(d-3c_n)}{2(c_0-v)^2-8w}$, $c^*_{R1}=\frac{v}{2}+\frac{c_0}{2}$, $p^*_{R2}=\frac{d(c0-v)^2+(-d-cn)w-2dw}{(c_0-v)^2-4w}$, and $r^*_R=\frac{(c_0-v)(d-c_n)}{2(c_0-v)^2-8w}$.

All the proofs are in Appendix A.

In Theorem 1, the decision equilibrium solutions of the manufacturers and the seller can be substituted into Formula (4), Formula (5) and Formula (6) to obtain the optimal decision profits of the three: $\pi^*_{R1}=-\frac{w(c_n-d)^2(c_0{}^2-c_1c_0-vc_0+c_1v-2w)}{2((c_0-v)^2-4w)^2}$, $\pi^*_{R2}=-\frac{w(c_n-d)^2}{2((c_0-v)^2-4w)}$, and $\pi^*_{R3}=\frac{w(c_0-v)(c_n-d)^2(3c_0-4c_1+v)}{8((c_0-v)^2-4w)^2}$.

Under the decentralized decision, there is a set of optimal prices for selling new product and repurchasing used power batteries for the power battery manufacturer. When the repurchasing price remains the same, increasing or decreasing the selling price of related products will always reduce the sales profit of the manufacturer, thus leading to the reduction of the overall profit obtained. Similarly, when the manufacturer sells new power battery products at the same price, increasing or decreasing the price of buying back the used power battery will reduce the income level. If the sales price of new products and the repurchase price of used products change at the same time, the overall profit of power battery manufacturers will be lower.

For the downstream seller, the price of purchasing the manufacturer's new power battery product is known. Under this premise, the seller has an optimal price selling to consumers to reach the maximum profit. Increasing or decreasing the price will reduce the seller’s sales profit, thus resulting in the reduction of the overall profit obtained by the enterprises.

Similarly, for the third-party recycler, the repurchase price of used power batteries determined by the power battery manufacturer will directly affect the recovery rate of used power battery products of the third-party recycler facing the consumer market. Under the

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premise that the price of used power batteries repurchased by manufacturers is given, the change of market recovery rate by the third-party recycler will reduce the overall income level of recycling enterprises.

2.3.2 Centralized decisions

In the third-party recycling channel under the state of centralized decision, all participants in the power battery closed-loop supply chain will become a whole. All participants seek the maximization of common interests rather than the maximization of each participant's own profit. According to the profit function of the battery manufacturer, the downstream seller and the third-party recycler in the previous section, the overall profit maximization model of the closed-loop supply chain is

$$\pi_R = \left[p_2 - c_n + (v - c_0)r\right](d - p_2) - wr^2/2, \tag{7}$$

where π_R represents the overall profit of the power battery closed-loop supply chain. $\left[p_2 - c_n + (v - c_0)r\right](d - p_2)$ represents the revenue gained by selling new products with part of the power batteries coming from remanufacturing, and $-wr^2/2$ represents the investment for recovering used power batteries in the closed-loop supply chain.

Through the expected profit function of Equation (7), the centralized optimal decisions of the power battery closed-loop supply chain under the recycling channel of the third-party recycler can be obtained.

Theorem 2. When $2w - (c_0 - v)^2 > 0$, denote p'^{*}_{R2} and r'^{*}_{R} as the downstream seller's optimal selling price and consumer market recovery rate respectively. Then the equilibrium solution in the recycling channel of the third-party recyclers under the centralized decision is $p'^{*}_{R2} = \dfrac{w(d + c_n) - d(v - c_0)^2}{2w - (v - c_0)^2}$ and $r'^{*}_{R} = \dfrac{(d - c_n)(v - c_0)}{2w - (v - c_0)^2}$.

Based on the equilibrium decision Theorem 2, we can determine the total profits of supply chain $\pi'^{*}_{R} = \dfrac{w(d - c_n)^2}{4w - 2(v - c_0)^2}$.

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For the third-party recycler recycling channel under centralized decision making, there is a group of optimal decisions of closed-loop supply chain participants, making the closed-loop supply chain achieving the maximization of the total profit. At this point, if the consumer market recovery rate holds still, the selling price of the change (either increase or decrease) will always lead to a drop in the closed-loop supply chain profit. Similarly, when the sales price remains unchanged, the fluctuations in the market recovery rate of used power batteries will reduce the overall profit of the closed-loop supply chain. To sum up, under the centralized decision, all participants in the whole power battery closed-loop supply chain jointly determine the optimal prices, which can maximize the overall interests of all participants. According to the above model, the sales price and market recovery rate of power battery for consumers can be obtained through the cooperation of participants in the closed-loop supply chain under centralized decision-making.

2.3.3 Comparison

Compare the results under decentralized and centralized decisions, we can have the following theorem.

Theorem 3. Compared with decentralized decision making, under centralized decision making (1) the selling price to consumers is higher; (2) the rate of recycling used power batteries from consumers is lower; and (3) the overall benefits of the closed-loop supply chain are higher.

According to Theorem 3, in the recycling channel of the third-party recycler, compared with the decentralized decision, the centralized decision can obtain a relatively higher selling price of power battery products facing consumers, and a relatively lower market recovery rate of used power battery products. Although the recycling rate of used power batteries is decreased, the total profit of the closed-loop supply chain will increase under the influence of the higher unit selling price set by the downstream seller. The profit generated by the sales of high-priced new products in the system exceeds the recovery income generated by the reverse logistics network at a higher recovery rate. Therefore, the total profit in the power battery closed-loop supply chain system is increased. It also shows that the centralized decision-making with multiple participants eliminates the dual marginal effect under the decentralized decision-making to a certain extent, and the centralized decision-making of the power battery closed-loop supply chain is more favorable to improve the overall benefit of the supply chain.

The good development of the power battery market plays a key role in the stability of the national new energy vehicle market and economic growth. However, the ensuing

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environmental pollution and social resource waste caused by the scrapping of power battery products make the government have to consider the effective recovery and utilization of the waste power battery products. In the next section, we will study the market of power battery closed-loop supply chain under the supervision of the government and how each participant formulates the optimal strategy according to the reward and punishment policies.

III. SUPPLY CHAIN COORDINATION WITH GOVERNMENT POLICY

In 2018, Chinese government issued the "New Energy Vehicle Power Battery Recycling Management Interim Measures", clearly stating that all power battery manufacturers and electric vehicle manufacturers carry main responsibility for the vehicle power battery recycling. Therefore, we model the manufacturer as the responsible subject to guide the recycling activities of the closed-loop supply chain to regulate the market. The government's incentives and punishments for the closed-loop supply chain are also mainly for power battery manufacturers.

Assume that the amount of reward (punishment) given by the government to the participating members of the closed-loop supply chain is $T = k(r - r_0)$, where k represents the reward and punishment intensity set by the government, and r_0 represents the target recovery rate of the used power battery market required by the government. $T > 0$ means that the product recovery rate of the power battery market exceeds the target recovery rate, and the manufacturer is rewarded with subsidies from the government. When $T < 0$, on the contrary, the government imposes a certain amount of punishment.

3.1 The Effect of The Government Policy

3.1.1 The government only rewards and punishes the manufacturer

Denote Q as the scenario where the government rewards and punishes only manufacturers. Then the expected revenue functions of the power battery manufacturer, the downstream seller and the third-party recyclers are respectively:

$$\pi_{QR1} = \left[p_1 - c_n + (v - c_1)r \right](d - p_2) + k(r - r_0)\,, \tag{8}$$

$$\pi_{QR2} = (p_2 - p_1)(d - p_2)\,, \tag{9}$$

$$\pi_{QR2} = (c_1 - c_0)r(d - p_2) - wr^2 / 2\,. \tag{10}$$

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The decentralized optimal decisions of each participant can be obtained as shown in the next theorem.

Theorem 4. The recycling channel of the third-party recycler includes three game players: the manufacturer, the downstream seller and the third-party recycler. Set p_{QR1} and c_{QR1} to represent the unit price of new products sold by the power battery manufacturer and the unit cost of purchasing used power batteries from the third-party recycler respectively. p_{QR2} represents the unit price of new products sold by the downstream seller to consumers, and r_{QR} represents the recovery rate of used power batteries. If there are only certain p^*_{QR1}, c^*_{QR1}, p^*_{QR2}, r^*_{QR} ,so that the expected revenue of the power battery manufacturer and the downstream seller reach their own maximums, then the optimal decisions of the manufacturer, the downstream seller and the third-party recycler are $p^*_{QR1} = \frac{(v-c_0)\left[(v-c_0)(d-c_n)+2k\right]}{4\left[(v-c_0)^2-4w\right]} + \frac{w(d+3c_n)-k(v-c_0)}{4w}$, $c^*_{QR1} = \frac{k\left[4w-(v-c_0)^2\right]}{2w(d-c_n)+k(v-c_0)} + \frac{v}{2} + \frac{c_0}{2}$, $p^*_{QR2} = \frac{2d(v-c_0)^2-2w(3d+c_n)+k(v-c_0)}{2\left[(v-c_0)^2-4w\right]}$, and $r^*_{QR} = \frac{-2w(v-c_0)(d-c_n)+k(v-c_0)^2-8kw}{4w\left[(v-c_0)^2-4w\right]}$.

With the decision equilibrium solutions of the manufacturer, the downstream seller and the third-party recycler in Theorem 4, the optimal decision profit of the participating members of the closed-loop supply chain can be obtained as $\pi^*_{QR1} = -\frac{\left[(v-c_0)(d-c_n)+2k\right]^2}{16\left[(v-c_0)^2-4w\right]} + \frac{w(d-c_n)^2+3k^2}{16w} - kr_0$, $\pi^*_{QR2} = -\frac{\left[2w(d-c_n)+k(v-c_0)\right]^2}{8w\left[(v-c_0)^2-4w\right]}$, and $\pi^*_{QR3} = -\frac{\left[2w(v-c_0)(d-c_n)-k(v-c_0)^2+8kw\right]^2}{32w\left[(v-c_0)^2-4w\right]}$.

3.1.2 The government rewards and punishes both the manufacturer and the down-stream seller

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The section focuses on the situation when the manufacturer and distributor sharing the responsibility. Denote this situation as W. As the main participating member of the closed-loop supply chain, the downstream seller obtains corresponding profits from the sales of products and also bears the responsibility of recycling part of used power batteries. Assume the portion taken by the power battery manufacturer is α ($0<\alpha<1$) and taken by the downstream seller is then $1-\alpha$. Thus, the profit function of each participant in the closed-loop supply chain can be obtained as follows:

$$\pi_{WR1}=\left[p_1-c_n+(v-c_1)r\right](d-p_2)+\alpha k(r-r_0), \tag{11}$$

$$\pi_{WR2}=(p_2-p_1)(d-p_2)+(1-\alpha)k(r-r_0), \tag{12}$$

$$\pi_{WR2}=(c_1-c_0)r(d-p_2)-wr^2/2. \tag{13}$$

Through maximizing the above expected profit function of each participant, the optimal decision of each participant in this scenario can be obtained.

Theorem 5. The recycling channel of the third-party recycler includes three game players: the power battery manufacturer, the downstream seller and the third-party recycler. Set p_{WR1} and c_{WR1} respectively represent the unit price of new products sold by the manufacturer and the unit cost of repurchasing used products from the third-party recycler. p_{WR2} represents the unit price of new products sold by the downstream seller to consumers, and r_{WR} represents the recovery rate of used power batteries. If there are only certain p^*_{QR1}, c^*_{QR1}, p^*_{QR2} and r^*_{QR}, so that the expected revenue of the power battery manufacturer and the downstream seller reach their own maximums, then the optimal decisions of the manufacturer, the seller and the recycler under the recycling channel of the third-party recycler are

$$p^*_{WR1}=\frac{(v-c_0)\left[(v-c_0)(d-c_n)+2k\right]}{4\left[(v-c_0)^2-4w\right]}+\frac{w(d+3c_n)+k(v-c_0)(1-2\alpha)}{4w},$$

$$c^*_{WR1}=\frac{k\alpha\left[4w-(v-c_0)^2\right]}{2w(d-c_n)+k(v-c_0)}+\frac{v}{2}+\frac{c_0}{2}, \quad p^*_{WR2}=\frac{2d(v-c_0)^2-2w(3d+c_n)+k(v-c_0)}{2\left[(v-c_0)^2-4w\right]}, \text{ and}$$

$$r^*_{WR}=\frac{-(v-c_0)(d-c_n)-4k\alpha}{2\left[(v-c_0)^2-4w\right]}-\frac{k(1-2\alpha)(v-c_0)^2}{4w\left[(v-c_0)^2-4w\right]}.$$

With the decision equilibrium solutions in Theorem 5 the optimal profits of the participating members can be obtained as

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$$\pi_{WR1}^{*}=-\frac{\left[(v-c_0)(d-c_n)+2k\right]^2}{16\left[(v-c_0)^2-4w\right]}+\frac{w(d-c_n)^2-k^2(1-4\alpha^2)}{16w}-k\alpha r_0 ,$$

$$\pi_{WR2}^{*}=-\frac{\left[2w(d-c_n)+k(v-c_0)\right]^2}{8w\left[(v-c_0)^2-4w\right]}+\frac{k^2\alpha(1-\alpha)}{2w}-k(1-\alpha)r_0 ,$$ and

$$\pi_{WR3}^{*}=-\frac{\left[2w(v-c_0)(d-c_n)+k(1-2\alpha)(v-c_0)^2+8kw\alpha\right]^2}{32w\left[(v-c_0)^2-4w\right]} .$$

3.1.3 The government rewards and punishes both the manufacturer and the third-party recycler

Since the third-party recycler plays an important role in the recycling of closed-loop supply chain system, it is also necessary to consider the case where the manufacturer and the third-party recycler state sharing the responsibility. Denote the scenario as E. Still use α to represent the portion of responsibility carried by the power battery manufacturer, and $1-\alpha$ to represent the portion shared by the third-party recycler. Then the profit functions of each participant are

$$\pi_{ER1}=\left[p_1-c_n+(v-c_1)r\right](d-p_2)+\alpha k(r-r_0) \tag{14}$$

$$\pi_{ER2}=(p_2-p_1)(d-p_2) \tag{15}$$

$$\pi_{ER2}=(c_1-c_0)r(d-p_2)-wr^2/2+(1-\alpha)k(r-r_0) \tag{16}$$

The optimal decisions of each participant in the scenario are shown in Theorem 6.

Theorem 6. In the recycling channel of the third-party recycler, set p_{ER1} and c_{ER1} respectively to represent the unit price of new products sold by the power battery manufacturer and the unit cost of purchasing used power batteries from the third-party recycler. p_{ER2} represents the unit price of new products sold by the downstream seller to consumers, and r_{ER} represents the recovery rate of used power batteries. If there are only certain $p_{ER1}^{*}, c_{ER1}^{*}, p_{ER2}^{*}$ and r_{ER}^{*} so that the expected revenue of the power battery manufacturer and the third-party recycler reach their own maximums, then the optimal decisions of the manufacturer, the downstream seller and the third-party recycler are

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$$p_{ER1}^{*}=\frac{(v-c_0)\left[(v-c_0)(d-c_n)+2k\right]}{4\left[(v-c_0)^2-4w\right]}+\frac{w(d+3c_n)-k(v-c_0)}{4w},$$

$$c_{ER1}^{*}=-\frac{k(1-2\alpha)\left[4w-(v-c_0)^2\right]}{2w(d-c_n)+k(v-c_0)}+\frac{v}{2}+\frac{c_0}{2}\ ,\quad p_{ER2}^{*}=\frac{2d(v-c_0)^2-2w(3d+c_n)+k(v-c_0)}{2\left[(v-c_0)^2-4w\right]}\ ,\ \text{and}$$

$$r_{ER}^{*}=\frac{k(v-c_0)^2-2w(v-c_0)(d-c_n)-8kw}{4w\left[(v-c_0)^2-4w\right]}.$$

Substituting the optimal decisions in Theorem 6 into Formula (14), Formula (15) and Formula (16) respectively, the optimal profits of the participating members can be obtained as

$$\pi_{ER1}^{*}=-\frac{\left[(v-c_0)(d-c_n)+2k\right]^2}{16\left[(v-c_0)^2-4w\right]}+\frac{w(d-c_n)^2-3k^2}{16w}-k\alpha r_0\ ,\quad \pi_{ER2}^{*}=-\frac{\left[2w(d-c_n)+k(v-c_0)\right]^2}{8w\left[(v-c_0)^2-4w\right]}\ ,$$

and $\pi_{ER3}^{*}=-\frac{\left[2w(v-c_0)(d-c_n)-k(v-c_0)^2+8kw\right]^2}{32w\left[(v-c_0)^2-4w\right]}-k(1-\alpha)r_0$.

3.1.4 Comparison

According to the analysis of the above three government rewards and punishments (Q, W and E), we can find the following propositions.

Proposition 1. $p_{QR2}^{*}=p_{WR2}^{*}=p_{ER2}^{*}$.

The selling price to consumers will not change under different government rewards and punishments. This means that the allocation of responsibility to the manufacturers, the downstream seller or the third-party recycler has no effect on consumer retailing price or market demand. And the change of the proportion of responsibility undertaken will not affect the pricing of power battery products. This is because if the downstream seller increases the price of the product, the consumer demand will decrease. Then due to the existence of the initial investment in recycling used power batteries, the reduction of demand damages the income of the recycler, leading to the reduction of the recycling initiative and the recovery rate cannot meet the recycling standards required by the policy. In this case, the penalty measures against the downstream seller become effective. Also, if the downstream seller reduces the price, it will lead to the reduction of their own revenue. Therefore, different from the industry alliance recycling channel, retailers under the third-party recycling channel do not easily

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change the retail price in the consumer market.

Proposition 2. $c_{QR1}^{*} > c_{WR1}^{*} > c_{ER1}^{*}$.

The highest repurchasing price of used power batteries exists in the situation where the government only applies incentives and punishments to the power battery manufacturer, followed by the situation where the manufacturer and the downstream seller share the responsibility. The situation where the manufacturer and the third-party recycler share the responsibility of recycling has the lowest recovery price. With independent responsibility, the power battery manufacturer chooses to set the highest recycling price to stimulate the enthusiasm of the third-party recycler and ultimately reduce the risk of government punishment. Therefore, when the power battery manufacturer undertakes the responsibility of recycling independently, the price of used power batteries sold to the manufacturer will be increased, so as to increase the market product recovery rate.

Proposition 3. $r_{QR}^{*} = r_{ER}^{*} > r_{WR}^{*}$.

The lowest market recovery rate happens when the manufacturer and distributor share the responsibility for the recycling of used power battery products. The analysis shows that the market recovery rate of waste power battery products increases gradually with the increase of government reward and punishment intensity. The reward and punishment policy of sharing responsibility between the manufacturer and the downstream seller does not directly affect the third-party recycler, so it is difficult to drive the enthusiasm of recycling in the closed-loop supply chain. Because the seller is not directly involved in the recycling of used power battery, the reward and punishment activities facing the seller are inert and cannot effectively improve the recovery level.

Proposition 4. $p_{WR1}^{*} > p_{QR1}^{*} = p_{ER1}^{*}$.

Proposition 4 shows that in the third-party recycler recycling channel, in the case the manufacturer shares responsibility independently and the case the manufacturer and the third-party recycler share responsibility together, the wholesale price is the same and lower than the other case. For the case sharing responsibility with the downstream seller, since high power battery recycling price may lead to the manufacturer receiving the punishment, so the manufacturer increases the wholesale price for the seller to increase the profit.

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Through the analysis, it can be found that the wholesale price, retail price and consumer market recovery rate of used power batteries are the same under the two situations where the power battery manufacturer fully taking the recycling responsibility and the manufacturer and the recycler share the recycling responsibility. The buyback price of used power batteries is the lowest when the manufacturer and the recycler share the recycling responsibility. And the manufacturer and the downstream seller sharing responsibility scenario has the lowest market rates of recovery and the highest wholesale price.

3.2 Coordination Mechanism Design Based on the Recovery Rate

3.2.1 Supply chain system coordination under Q and E

Note that under the centralized decision of the recycling channel of the third-party recycler, the recovery rate of the used power battery in the system exists $r'^{*}_{R} = \frac{(d-c_n)(v-c_0)}{2w-(v-c_0)^2}$ and the market recovery rate is $r^{*}_{QR} = r^{*}_{ER}$. Therefore, in order to achieve the optimal effect of centralized decision-making state for the product recovery rate using the government's policy, and to complete the coordination of the supply chain system under the consumer market recovery rate, the condition $r^{*}_{QR} = r'^{*}_{R}$ need to be satisfied.

After being brought in, we can get the reward and punishment intensity after the government coordinated the closed-loop supply chain of power battery market as $k^{*}_{QR} = -\frac{2w(v-c_0)(d-c_n)\left[(v-c_0)^2-6w\right]}{\left[(v-c_0)^2-8w\right]\left[(v-c_0)^2-2w\right]}$.

Thus, the system recovery rate of the power battery manufacturer can be improved by Pareto if the government rewards and punishments the power battery manufacturer with the reward and punishment intensity equal to k^{*}_{QR}.

Because of $\frac{\partial k^{*}_{QR}}{\partial w} < 0$, the difficulty of power battery product recycling in the closed-loop supply chain market increases, which increases the strength of the government's reward and punishment force against the manufacturer. This is consistent with intuition. The increasing difficulty of recycling used power batteries in the consumer market will lead to the increase of related recycling costs and the reduction of overall benefits. As the supervisor, the government

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will reduce the intensity of rewards and punishments.

Let $\delta = v - c_0$ and put into the closed-loop supply chain system coordinated by the government. The reward and punishment intensity can be obtained as $k_{GR}^* = -\frac{2w\delta(d-c_n)(\delta^2-6w)}{(\delta^2-8w)(\delta^2-2w)}$.

The partial derivative of this function with respect to δ is $\frac{\partial k_{GR}^*}{\partial \delta} > 0$. As the remanufacturing cost advantage of waste power battery increases, the subsidy (penalty) intensity of the government increases at the same time. In this case, the government can be stimulated to guide the recycling of the used power batteries. Similarly, the improvement of the remanufacturing cost advantage can motivate the battery manufacturer to repurchase the recycled used power batteries, promote the circulation of remanufactured products in the closed-loop supply chain system, and realize the decision optimization of the manufacturer's recycling responsibility system under the government reward and punishment mechanism.

3.2.2 Supply chain system coordination under W

Note that the market recovery rate of case W is $r_{WR}^* = \frac{-(v-c_0)(d-c_n)-4k\alpha}{2\left[(v-c_0)^2-4w\right]} - \frac{k(1-2\alpha)(v-c_0)^2}{4w\left[(v-c_0)^2-4w\right]}$.

To make the system recovery rate meet the optimal effect under the centralized decision, the closed-loop supply chain coordination based on the consumer market recovery rate is necessary. The following conditions need to be satisfied $r_{WR}^* = r'^*_R$.

Then we can get the reward and punishment intensity after the government coordinated the closed-loop supply chain of power battery market is $k_{WR}^* = \frac{2w(v-c_0)(d-c_n)\left[(v-c_0)^2-6w\right]}{\left[(v-c_0)^2-2w\right]\left[8w\alpha+(1-2\alpha)(v-c_0)^2\right]}$.

Since $\frac{\partial k_{WR}^*}{\partial w} < 0$, we can get that with the increasing difficulty in recycling waste power battery products in the closed-loop supply chain recycling market, the intensity of reward and

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punishment imposed on the manufacturer by the government will decrease in order to ensure the effective operation of the system. When the consumer market is more difficult to recover power battery batteries, related recovery costs increase and overall profits decrease, the government, as a regulator, will reduce the amount of investment. That is, the intensity of rewards and punishments will be weakened. As $\frac{\partial k_{WR}^*}{\partial \alpha} < 0$, if the seller's recycling responsibility is reduced, the intensity of rewards and punishments will be reduced at the same time.

Set $\delta = v - c_0$ and bring into the closed-loop supply chain system coordinated by the government. The rewards and punishments can be obtained as $k_{WR}^* = -\frac{2w\delta(d-c_n)(\delta^2-6w)}{(\delta^2-2w)[(1-2\alpha)\delta^2-8w\alpha]}$.

The partial derivative of this function with respect to δ is $\frac{\partial k_{WR}^*}{\partial t} > 0$. With the increase of the remanufacturing cost advantage of waste power battery, the intensity of government subsidy (penalty) also increases. This can stimulate the increase of the closed-loop supply chain system of lead products. Also, the improvement of remanufacturing cost advantage can motive the battery manufacturer to repurchase the used power batteries and promote the development of the closed-loop supply chain system.

IV. NUMERICAL ANALYSIS

In order to study the influence of the government's policy on the decision-making behavior of the participants in the power battery closed-loop supply chain and related coordination mechanism, this section sets relevant parameters based on the research and analysis of the Chinese market and uses the simulation program Matlab_R2016a and Maple_2020. The effect of government incentives and punishments on the behaviors of the closed-loop supply chain subjects and the Pareto improvement of the system is studied.

Set $c_0 = 10, c_n = 50, v = 30, k = 1200, d = 100, r_0 = 0.4$ and $w = 800$. With the help of the simulation software, the trend chart of the revenue of the power battery manufacturer, the downstream seller and the third-party recycler with the government's participation of rewards and punishments in the closed-loop supply chain under the decentralized decision-making state can be obtained as follows.

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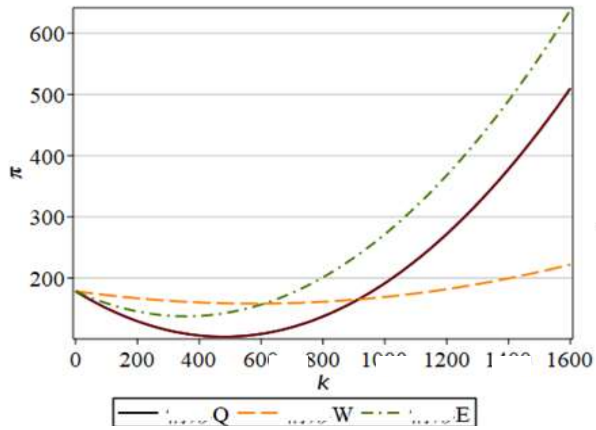

Fig 2: The manufacturer's profit changing with k

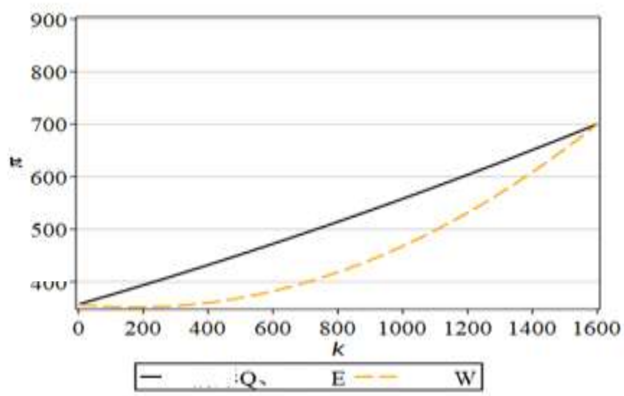

Fig 3: The downstream seller's profit changing with k

Referring to Figure 2, the manufacturer's profit in the three states decrease first and then increase with the increase of government rewards and punishments. In case E, the manufacturer's profit changes the fastest, and the government's reward and punishment mechanism has the greatest impact on the marginal benefit, followed by case Q, and finally case W. In addition, the profit of the power battery manufacturer in the three scenarios will be less than that in the case of no reward or punishment.

It can be seen from Figure 3 that Q and E in the recycling channel have the same profit level for the downstream seller, which increases with the increase of government rewards and punishments, showing a positive correlation. In addition, when the seller shares the recycling responsibility, that is, the profit of the seller in scenario W decreases first and then increases

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with the increase in the intensity of government incentives and punishments. In the initial stage, the profit of the seller in this scenario is lower than the other two situations, but when the intensity of government incentives and punishments increases to a certain level, the profit level in scenario W exceeds that in scenario Q and E.

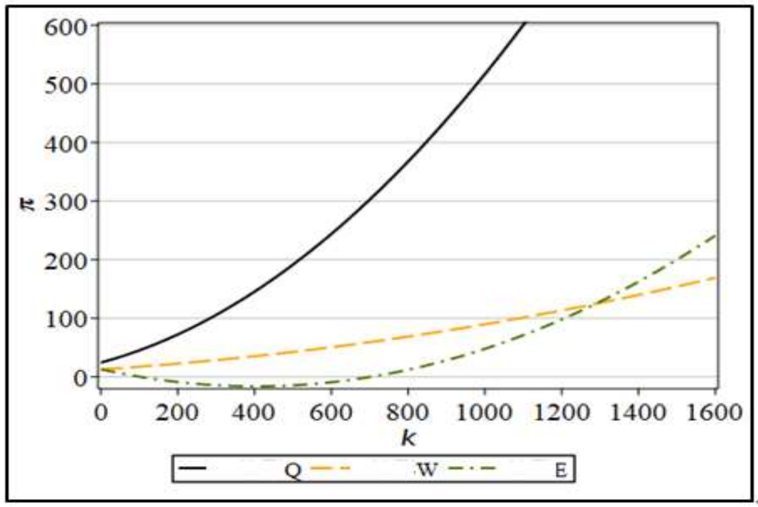

Fig 4: The third-party recycler’s profit changing with k

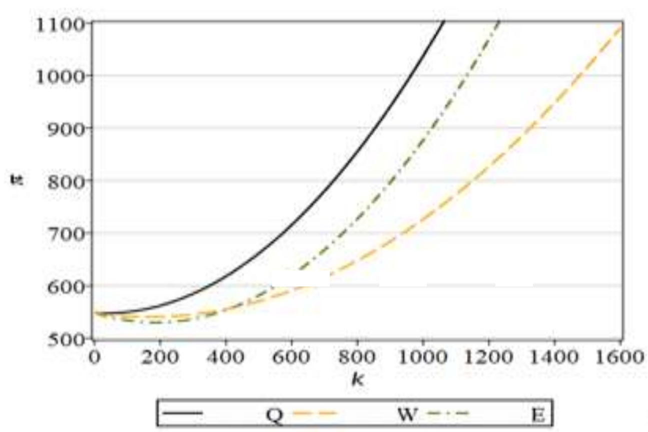

Fig 5: The total profit of supply chain changing with k

According to Figure 4, when the power battery manufacturer undertakes the recycling responsibility alone, the profit gained by the third-party recycler reaches the highest level no matter how the government's reward and punishment intensity changes. The profit of the third-party recyclers in cases Q and W both increases with the increase of the intensity of

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government rewards and punishments. But under the situation E, government rewards and punishment strength first lead to a negative yield of the third-party recycler. And when the rewards and punishment strength keep increasing, the third-party recycler's profit will become positive and finally exceed t the profit in situation W.

According to the Figure 2 and Figure 4, if the power battery manufacturer is re-sponsible for recycling independently, the manufacturer will suffer losses. The other two members in the closed-loop supply chain will become "free-riders". The rewards and punishment policy implemented by the government reduces the benefits of the manufacturer, but increases the profits of other members. The whole closed-loop supply chain system is improved by Pareto.

The overall profit of the closed-loop supply chain system is shown in Figure 5. With the gradual increase of government reward and punishment intensity, the total profit level of power battery closed-loop supply chain decreases first and then increases. Moreover, the marginal revenue of the supply chain system gradually increases. Greater intensity of rewards and punishments brings more benefit for the system. Also, the total system profit in the case where only the manufacturer bears the responsibility for recovery is always higher than that in the other two cases. When the downstream seller or the third-party recycler is required to share the responsibility, scenario W can obtain higher total revenue when the government rewards and punishments intensity is relatively small, while scenario E is more appropriate when the intensity is relatively large.

The profits of each member changing with the portion of responsibility taken by the manufacturer are shown in Figure 6 to 8.

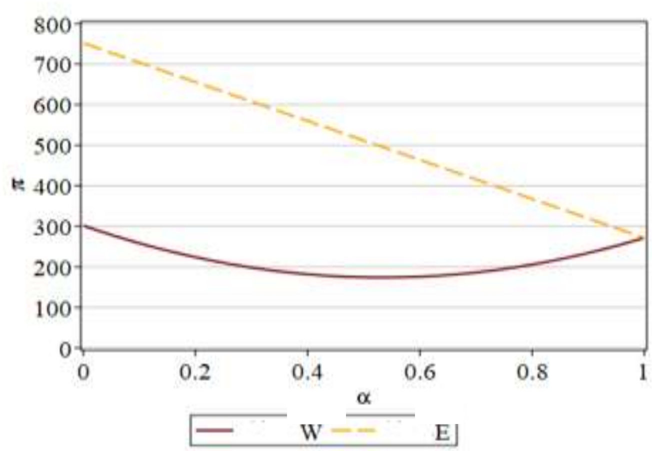

Fig 6: The manufacturer's profit changing with α

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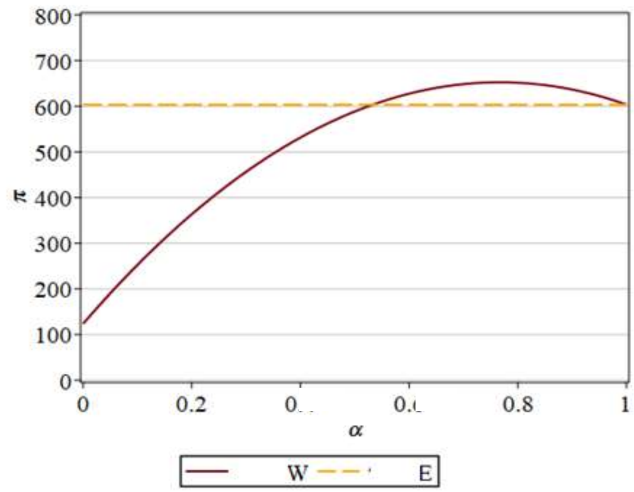

Fig 7: The downstream seller's profit changing with α

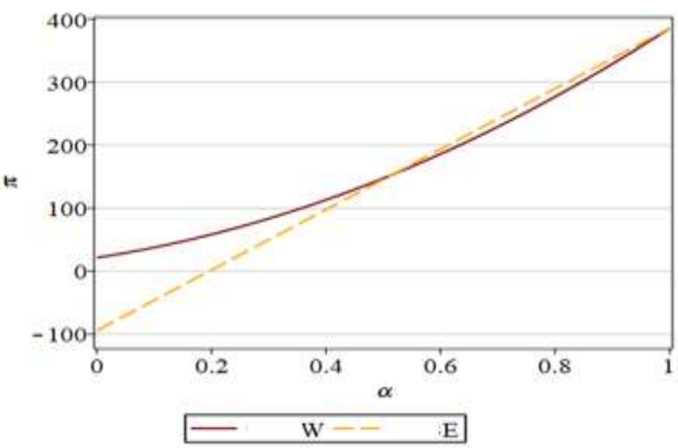

Fig 8: The third-party recycler's profit changing with α

Referring to Figure 6, it can be seen that with the increase in the portion of recycling responsibility taken by the power battery manufacturer, the manufacturer's profit level in scenario W first decreases and then increases. While in scenario E, it is always inversely proportional to the proportion of responsibility sharing. When the portion becomes 1, it will be the state of Scenario Q. Figure 7 shows that in Scenario E, no matter how the por-tion of responsibility sharing changes, the downstream seller's profit will not be affected. However, under scenario W, the profit of the downstream seller increases first and then decreases with

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the decrease of their own responsibility sharing ratio. The profit of the seller will exceed that in the case of no responsibility taken when the responsibility sharing ratio drops to a certain level. It can be seen from Figure 8 that the profit of the third-party recycler in both case W and case E increases with the increase of the respon-sibility sharing ratio of the power battery manufacturer. And there exists a certain level of responsibility taken by the manufacturer so that when the ratio is lower than the level, the third party recycler's profit in situation W is higher and vice versa. It is worth noting that in case E, when the third-party recycler takes too much recycling responsibility, the profit will drop to be negative.

V. CONCLUSION

Based on the theory of Stackelberg game, the closed-loop supply chain with one power battery manufacturer, one downstream seller and one third-party recycler in presence of recycling responsibility sharing is studied. And the influence of the govern-ment's reward and punishment policy on each member is analyzed. We find that regardless of the responsibility sharing and the government's policy, the retail price of the power battery products remain the same. And in the cases where the manufacturer takes all the recycling responsibility and the manufacturer sharing with the third-party recycler, the wholesale price and market recovery rate are the same under the government's pol-icy.

When the recycling responsibility is required to taken by the manufacturer, the wholesale price of new products decreases and the manufacturer's profit is reduced. But both the market recovery rate and the overall profit of the supply chain system increase. When the downstream seller shares the recycling responsibility with the manufacture, his own profit reaches the lowest level among all the cases. And in this case, the excessive intensity of rewards and punishments will lead to the reduction of the overall profit of all participants. When the third-party recycler shares the recycling responsibility, both the wholesale price and the recycling price are smaller while the market recovery rates is higher. Comparing the effects of the government's policy, we find that for the situation where the manufacturer and the third-party recycler share the recycling responsibility, relatively low intensity of rewards and punishments shows better performance. Mean-while, relatively high intensity is more for the situation of sharing the responsibility with the downstream seller.

The research on recycling of closed-loop supply chain is an important field. The problem of government reward and punishment mechanism under different recycling channels is related to the direct interests of upstream and downstream enterprises in the supply chain. This paper improves the theory of closed-loop supply chain and provides some suggestions for the enterprises in the electric vehicle battery market. Although the research has achieved some meaningful results, there are still some shortcomings. For example, each stage in the closed-loop supply chain only involves one participating enterprise while more than one enterprise grabs the market share together in practice. Also, there are many cases of the coexistence of the two channels. The supply chain decision making problems under the coexistence of the recycling channel of both the indus-trial alliance and the recycling channel of the third-party recycler is worth studied in the future.

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Appendix A

Proof of Theorem 1. Solve the first-order and second-order partial derivatives of Formula (6) on the recovery rate of used power batteries recycled by the recycler, we have

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$\frac{\partial \pi_2}{\partial r} = (c_1 - c_0)(a - p_2) - rw; \frac{\partial^2 \pi_1}{\partial^2 r} = -w < 0$. Therefore π_{R2} is concave in r and has a unique maximum. Let $\frac{\partial \pi_{R2}}{\partial r} = 0$, we have $r = \frac{(c_1 - c_0)(d - p_2)}{w}$. Substitute $r = \frac{(c_1 - c_0)(d - p_2)}{w}$ and $p_2 = e + p_1$ into Equation (4). Then we can obtain the function expression of the profit of power battery manufacturer only with respect to the sales price p_1 and recovery price c_1 to consumers: $\pi_{R1} = \left[(p_1 - c_n) + \frac{(v - c_1)(c_1 - c_0)(d - e - p_1)}{w} \right] (d - e - p_1)$.

The first partial derivative of Formula π_{R1} with respect to variables p_1 and c_1 can be obtained as follows:

$$\frac{\partial \pi_{R1}}{\partial p_1} = c_n - p_1 - \left[\frac{(c_1 - c_0)(c_1 - v)}{w} + 1 \right] (e - d + p_1) - \frac{(c_1 - c_0)(c_1 - v)(e - d + p_1)}{w};$$

$$\frac{\partial \pi_{R1}}{\partial c_1} = \frac{(e - d + p_1)^2 (c_0 - 2c_1 + v)}{w}.$$

Let the partial derivative be equal to 0. When the market demand for power battery is not zero, note that $e = p_2 - p_1$. Then we can have $p_1 = d - p_2 + c_n - \frac{2(c_0 - c_1)(d - p_2)(c_1 - v)}{w}$ and $c_1 = \frac{v}{2} + \frac{c_0}{2}$. Substitute c_1 into p_1, then $p_1 = -\frac{(v - c_0)^2 (d - p_2)}{2w} + d - p_2 + c_n$. Substitute the values p_1, c_1 and r into Equation (5), and take the partial derivative with respect to p_2. Then we can have the optimal price $p_2^* = \frac{d(c0 - v)^2 + (-d - cn)w - 2dw}{(c_0 - v)^2 - 4w}$.

Substitute p_2^* into the expressions of r, p_1 and c_1 respectively, the optimal decision choices can be obtained as the following: $p_1^* = \frac{(d + c_n)(c_0 - v)^2 - 2w(d - 3c_n)}{2(c_0 - v)^2 - 8w}$; $c_1^* = \frac{v}{2} + \frac{c_0}{2}$; and $r^* = \frac{(c_0 - v)(d - cn)}{2(c_0 - v)^2 - 8w}$.□

Proof of Theorem 2. Solve the first-order and second-order partial derivatives of Formula (7) with respect to the seller's sales price p_2 of the battery product of new power, namely:

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$\frac{\partial \pi_R}{\partial p_2} = c_n + d - 2p_2 + r(c_0 - v)$, $\frac{\partial^2 \pi_R}{\partial^2 p_2} = -2 < 0$.

To find the first and second partial derivatives of the recycling rate r of used power batteries recycled by distributors, namely: $\frac{\partial \pi_R}{\partial r} = (v - c_0)(d - p_2) - rw$, $\frac{\partial^2 \pi_R}{\partial^2 r} = -w < 0$.

Then the Hesse matrix of variables p_2 and r is $H = \begin{bmatrix} -2 & c_0 - v \\ c_0 - v & -w \end{bmatrix}$.

Since $H_{11} = -2 < 0$, $\det(H) = 2w - (c_0 - v)^2$, when $2w - (c_0 - v)^2 > 0$, matrix H is negative definite matrix. Under this condition, the overall profit of the closed-loop supply chain π_R is the joint concave function of the retailer regarding the retail price p_2 and the recovery rate r of its consumer-facing products. And there exists a unique and determined value that makes π_R maximum in this case.

Referring to the first-order condition of the formula, let $\frac{\partial \pi_R}{\partial p_2} = 0$, $\frac{\partial \pi_R}{\partial r} = 0$, and we can get $p_2 = \frac{c_n + d + r(c_0 - v)}{2}$, $r = \frac{(v - c_0)(d - p_2)}{w}$. The equations can be simplified as $p_2 = \frac{w(d + c_n) - d(v - c_0)^2}{2w - (v - c_0)^2}$ and $r = \frac{(a - c_n)(v - c_0)}{2w - (v - c_0)^2}$.

Let p'^{*}_2 and r'^{*} respectively represent the unique values of the above p_2 and r, which are the optimal centralized decisions.□

Proof of Theorem 3. Since $2w - (c_0 - v)^2 > 0$, then there is

$$p'^{*}_{R2} - p^{*}_{R2} = \frac{d(v - c_0)^2 - w(d + c_n)}{(v - c_0)^2 - 2w} - \frac{d(c0 - v)^2 - w(3d + cn)}{(c_0 - v)^2 - 4w} > 0;$$

$$r'^{*}_R - r^{*}_R = \frac{(c_0 - v)(d - c_n)}{(v - c_0)^2 - 2w} - \frac{(c_0 - v)(d - c_n)}{2(c_0 - v)^2 - 8w} < 0;$$

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$$\begin{aligned}\pi'^{*}_{R2}-\pi^{*}_{R2} &= \pi'^{*}_{U}-\pi^{*}_{R1}-\pi^{*}_{R2}-\pi^{*}_{R3}\\ &= \frac{w(d-c_n)^2}{4w-2(v-c_0)^2}+\frac{w(c_n-d)^2(c_0{}^2-c_1c_0-vc_0+c_1v-2w)}{2((c_0-v)^2-4w)^2}\\ &+\frac{w(c_n-d)^2}{2((c_0-v)^2-4w)}-\frac{w(c_0-v)(c_n-d)^2(3c_0-4c_1+v)}{8((c_0-v)^2-4w)^2}\\ &>0\end{aligned} .\square$$

Proof of Theorem 4. Apply backward induction to analyze the problem. Solve Equation (10) for the first-order and second-order partial derivatives of market recovery rate r of used power batteries recycled by third-party recyclers, namely: $\frac{\partial \pi_{QR2}}{\partial r}=(c_1-c_0)(a-p_2)-rw$, $\frac{\partial^2 \pi_{QR2}}{\partial^2 r}=-w<0$. Then we know that π_{QR2} is concave in r and has a maximum. Let $\frac{\partial \pi_{QR2}}{\partial r}=0 \rightarrow r=\frac{(c_1-c_0)(d-p_2)}{w}$.

By substituting $r=\frac{(c_1-c_0)(d-p_2)}{w}$ and $p_2=e+p_1$, we can get the function expression of the profit of power battery manufacturer only with respect to the sales price p_1 and recovery price c_1 for consumers $\pi_{QR1}=\left[(p_1-c_n)+\frac{(v-c_1)(c_1-c_0)(d-e-p_1)}{w}\right](d-e-p_1)-k\left[r_0-\frac{(c_0-c_1)(e-d+p_1)}{w}\right]$.

In this case, the first partial derivative of Formula π_{QR1} with respect to variables p_1 and c_1 can be obtained as follows:

$$\frac{\partial \pi_{QR1}}{\partial p_1}=c_n-p_1+\left[\frac{(c_1-c_0)(v-c_0)}{w}-1\right](e-d+p_1)+\frac{(c_0-c_1)(c_0-v)(e-d+p_1)+k(c_0-c_1)}{w};$$

$$\frac{\partial \pi_{QR1}}{\partial c_1}=-\frac{(e-d+p_1)^2(c_0-v)+k(e-d+p_1)}{w}.$$

And let the partial derivative be equal to 0. When the market demand for power battery is not zero, put in $e=p_2-p_1$ to get $p_1=d-p_2+c_n-\frac{2(c_0-c_1)(d-p_2)(c_1-v)}{w}$ and

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$c_1=\frac{v}{2}+\frac{c_0}{2}+\frac{k}{2(d-p_2)}$. Substitute c_1 into p_1 to get the expression containing p_2, then $p_1=-\frac{(v-c_0)^2(d-p_2)}{2w}+d-p_2+c_n$. Substitute the values p_1, c_1 and r into Equation (9), and take its partial derivative with respect to p_2. Then we can have $p_2^*=\frac{2d(v-c_0)^2-2w(3d+c_n)+k(v-c_0)}{2\left[(v-c_0)^2-4w\right]}$.

Finally, by substituting Equation p_2^* into the expressions of r, p_1 and c_1 respectively, the optimal decision choices of the manufacturer and the downstream seller in the power battery closed-loop supply chain with decentralized decisions regarding r, p_1 and c_1 can be obtained, namely:

$$p_1^*=\frac{(v-c_0)\left[(v-c_0)(d-c_n)+2k\right]}{4\left[(v-c_0)^2-4w\right]}+\frac{w(d+3c_n)-k(v-c_0)}{4w};$$

$$c_1^*=\frac{k\left[4w-(v-c_0)^2\right]}{2w(d-c_n)+k(v-c_0)}+\frac{v}{2}+\frac{c_0}{2};$$

$$r^*=\frac{-2w(v-c_0)(d-c_n)+k(v-c_0)^2-8kw}{4w\left[(v-c_0)^2-4w\right]}.$$

Proof of Theorem 5. The proof is similar to Theorem 4, so it is omitted here.

Proof of Theorem 6. The proof is similar to Theorem 4, so it is omitted here.

Proof of Proposition 1. Comparing the results in Theorem 4, 5 and 6, $p_{QR2}^*=p_{WR2}^*=p_{ER2}^*$ can be easily obtained.

Proof of Proposition 2. The subtraction of the two equations is $c_{QR1}^*-c_{WR1}^*=-\frac{2k(1-\alpha)[(v-c_0)^2-4w]}{2w(d-c_n)+k(v-c_0)}$. Given inequality $d-c_n>0$ and $v-c_0>0$, the above equation satisfies $2w(d-c_n)+k(v-c_0)>0$. Since $2w-(c_0-v)^2>0$ and $0<\alpha<1$, we have $2k(1-\alpha)[(v-c_0)^2-4w]<0$. So $c_{QR1}^*-c_{WR1}^*>0$ is true, which is $c_{QR1}^*>c_{WR1}^*$. Similarly, we can

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have $c_{WR1}^* > c_{ER1}^*$, and then $c_{QR1}^* > c_{WR1}^* > c_{ER1}^*$.

Proof of Proposition 3. Since $r_{ER}^* - r_{WR}^* = -\frac{k(1-\alpha)}{2w}$ and $0 < 1-\alpha < 1$, we have $r_{ER}^* - r_{WR}^* > 0$. And comparing the results in Theorem 4 and 6, $r_{QR}^* = r_{ER}^*$ can be obtained. Then it can be concluded that $r_{QR}^* = r_{ER}^* > r_{WR}^*$.

Proof of Proposition 4. Subtract the two equations to get $p_{WR1}^* - p_{QR1}^* = \frac{k(v-c_0)(1-\alpha)}{2w}$. Given the probability $0 < \alpha < 1$ and $v - c_0 > 0$, we can have $p_{WR1}^* - p_{QR1}^* = \frac{k(v-c_0)(1-\alpha)}{2w} > 0$. And $p_{QR1}^* = p_{ER1}^*$ can be obtained from the results in Theorem 4 and 6. In summary, $p_{WR1}^* > p_{QR1}^* = p_{ER1}^*$.